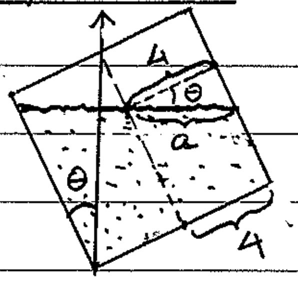
UKENS UTFORDRING № 4:

Challenging Problems, #1, s. 516.

LØSNING:



Det innsees lett at siden glasset er mer enn halv- fullt og det ikke renner over kanten, så blir ingen del av bunnen tørrlagt. (Tenk over dette!)

Vi antar som kjent at vannoverflaten da har form av en ellipse(*) der den korte halvaksen er 4 cm. Vi betegner den store halvakse a:

$\cos\theta = 4/a$ eller $a = 4/\cos\theta$

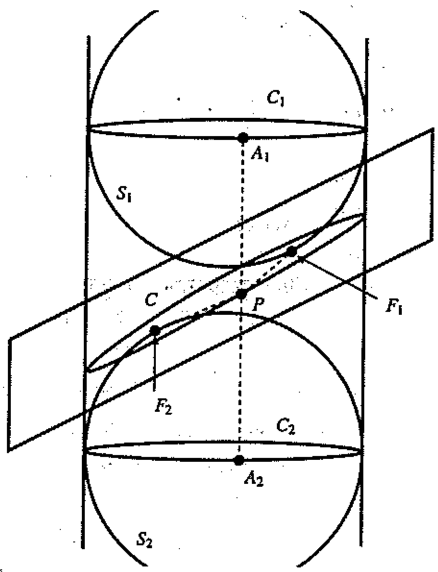
Ut fra Eks. 3, s. 503, blir arealet fölgelig:

$$A = \pi \cdot 4 \cdot a = \underline{16\pi/\cos\theta}$$

BEGRUNNELSE FOR PÅSTANDEN (*):

Dette er bevist som oppgave 2, Challenging Problems, s. 517

Challenging Problems 8 (page 516)



2. Let S_1 and S_2 be two spheres inscribed in the cylinder, one on each side of the plane that intersects the cylinder in the curve C that we are trying to show is an ellipse. Let the spheres be tangent to the cylinder around the circles C_1 and C_2, and suppose they are also tangent to the plane at the points F_1 and F_2, respectively, as shown in the figure.

Let P be any point on C. Let A_1A_2 be the line through P that lies on the cylinder, with A_1 on C_1 and A_2 on C_2. Then $PF_1 = PA_1$ because both lengths are of tangents drawn to the sphere S_1 from the same exterior point P. Similarly, $PF_2 = PA_2$. Hence

$$PF_1 + PF_2 = PA_1 + PA_2 = A_1A_2,$$

which is constant, the distance between the centres of the two spheres. Thus C must be an ellipse, with foci at F_1 and F_2.