EXERCISES 18.2

See Section 7.9 for exercises on separable equations and linear equations.

Solve the homogeneous differential equations in Exercises 1–6.

1. $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

2. $\dfrac{dy}{dx} = \dfrac{xy}{x^2+2y^2}$

3. $\dfrac{dy}{dx} = \dfrac{x^2+xy+y^2}{x^2}$

4. $\dfrac{dy}{dx} = \dfrac{x^3+3xy^2}{3x^2y+y^3}$

5. $x\dfrac{dy}{dx} = y + x\cos^2\left(\dfrac{y}{x}\right)$

6. $\dfrac{dy}{dx} = \dfrac{y}{x} - e^{-y/x}$

7. Find an equation of the curve in the xy-plane that passes through the point $(2, 3)$ and has, at every point (x, y) on it, slope $2x/(1+y^2)$.

8. Repeat Exercise 7 for the point $(1, 3)$ and slope $1 + (2y/x)$.

9. Show that the change of variables $\xi = x - x_0$, $\eta = y - y_0$ transforms the equation

$$\frac{dy}{dx} = \frac{ax+by+c}{ex+fy+g}$$

into the homogeneous equation

$$\frac{d\eta}{d\xi} = \frac{a\xi+b\eta}{e\xi+f\eta},$$

provided (x_0, y_0) is the solution of the system

$$ax + by + c = 0$$
$$ex + fy + g = 0.$$

10. Use the technique of Exercise 9 to solve the equation
$$\frac{dy}{dx} = \frac{x + 2y - 4}{2x - y - 3}.$$

Show that the DEs in Exercises 11–14 are exact, and solve them.

11. $(xy^2 + y)\,dx + (x^2y + x)\,dy = 0$

12. $(e^x \sin y + 2x)\,dx + (e^x \cos y + 2y)\,dy = 0$

13. $e^{xy}(1 + xy)\,dx + x^2e^{xy}\,dy = 0$

14. $\left(2x + 1 - \frac{y^2}{x^2}\right) dx + \frac{2y}{x}\,dy = 0$

Show that the DEs in Exercises 15–16 admit integrating factors that are functions of x alone. Then solve the equations.

15. $(x^2 + 2y)\,dx - x\,dy = 0$

16. $(xe^x + x\ln y + y)\,dx + \left(\frac{x^2}{y} + x\ln x + x\sin y\right) dy = 0$

17. What condition must the coefficients $M(x, y)$ and $N(x, y)$ satisfy if the equation $M\,dx + N\,dy = 0$ is to have an integrating factor of the form $\mu(y)$, and what DE must the integrating factor satisfy?

18. Find an integrating factor of the form $\mu(y)$ for the equation

$$2y^2(x + y^2)\,dx + xy(x + 6y^2)\,dy = 0,$$

and hence solve the equation. *Hint:* See Exercise 17.

19. Find an integrating factor of the form $\mu(y)$ for the equation $y\,dx - (2x + y^3e^y)\,dy = 0$, and hence solve the equation. *Hint:* See Exercise 17.

20. What condition must the coefficients $M(x, y)$ and $N(x, y)$ satisfy if the equation $M\,dx + N\,dy = 0$ is to have an integrating factor of the form $\mu(xy)$, and what DE must the integrating factor satisfy?

21. Find an integrating factor of the form $\mu(xy)$ for the equation

$$\left(x\cos x + \frac{y^2}{x}\right) dx - \left(\frac{x\sin x}{y} + y\right) dy = 0,$$

and hence solve the equation. *Hint:* See Exercise 20.