EXERCISES 18.5

Exercises involving the solution of second-order, linear, homogeneous equations with constant coefficients can be found at the end of Section 3.7.

Find general solutions of the DEs in Exercises 1–4.

1. $y''' - 4y'' + 3y' = 0$

2. $y^{(4)} - 2y'' + y = 0$

3. $y^{(4)} + 2y'' + y = 0$

4. $y^{(4)} + 4y^{(3)} + 6y'' + 4y' + y = 0$

5. Show that $y = e^{2t}$ is a solution of

$$y''' - 2y' - 4y = 0$$

(where $'$ denotes d/dt), and find the general solution of this DE.

6. Write the general solution of the linear, constant-coefficient DE having auxiliary equation $(r^2 - r - 2)^2(r^2 - 4)^2 = 0$.

Find general solutions to the Euler equations in Exercises 7–12.

7. $x^2y'' - xy' + y = 0$

8. $x^2y'' - xy' - 3y = 0$

9. $x^2y'' + xy' - y = 0$

10. $x^2y'' - xy' + 5y = 0$

11. $x^2y'' + xy' = 0$

12. $x^2y'' + xy' + y = 0$

13. Solve the DE $x^3y''' + xy' - y = 0$ in the interval $x > 0$.

14. Show that the change of variables $x = e^t$, $z(t) = y(e^t)$, transforms the Euler equation

$$ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$$

into the constant-coefficient equation

$$a\frac{d^2z}{dt^2} + (b - a)\frac{dz}{dt} + cz = 0.$$

15. Use the transformation $x = e^t$ of the previous exercise to solve the Euler equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0, \qquad (x > 0).$$