EXERCISES 18.8

1. Find the general solution of $y'' = (x-1)^2 y$ in the form of a power series $y = \sum_{n=0}^{\infty} a_n (x-1)^n$.
2. Find the general solution of $y'' = xy$ in the form of a power series $y = \sum_{n=0}^{\infty} a_n x^n$ with a_0 and a_1 arbitrary.
3. Solve the initial-value problem

$$\begin{cases} y'' + xy' + 2y = 0 \\ y(0) = 1 \\ y'(0) = 2. \end{cases}$$

4. Find the solution of $y'' + xy' + y = 0$ that satisfies $y(0) = 1$ and $y'(0) = 0$.
5. Find the first three nonzero terms in a power series solution in powers of x for the initial-value problem $y'' + (\sin x)y = 0$, $y(0) = 1$, $y'(0) = 0$.
6. Find the solution, in powers of x, for the initial-value problem

$$(1-x^2)y'' - xy' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

7. Find two power series solutions in powers of x for $3xy'' + 2y' + y = 0$.
8. Find one power series solution for the Bessel equation of order $\nu = 0$, that is, the equation $xy'' + y' + xy = 0$.