EXERCISES 9.5

Determine the centre, radius, and interval of convergence of each of the power series in Exercises 1–8.

1. $\sum_{n=0}^{\infty} \frac{x^{2n}}{\sqrt{n+1}}$

2. $\sum_{n=0}^{\infty} 3n\,(x+1)^n$

3. $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x+2}{2}\right)^n$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 2^{2n}}\, x^n$

5. $\sum_{n=0}^{\infty} n^3(2x-3)^n$

6. $\sum_{n=1}^{\infty} \frac{e^n}{n^3}\,(4-x)^n$

7. $\sum_{n=0}^{\infty} \frac{(1+5^n)}{n!}\, x^n$

8. $\sum_{n=1}^{\infty} \frac{(4x-1)^n}{n^n}$

9. Use multiplication of series to find a power series representation of $1/(1-x)^3$ valid in the interval $(-1, 1)$.

10. Determine the Cauchy product of the series $1 + x + x^2 + x^3 + \cdots$ and $1 - x + x^2 - x^3 + \cdots$. On what interval and to what function does the product series converge?

11. Determine the power series expansion of $1/(1-x)^2$ by formally dividing $1 - 2x + x^2$ into 1.

Starting with the power series representation

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots, \qquad (-1 < x < 1),$$

determine power series representations for the functions indicated in Exercises 12–20. On what interval is each representation valid?

12. $\frac{1}{2-x}$ in powers of x

13. $\frac{1}{(2-x)^2}$ in powers of x

14. $\frac{1}{1+2x}$ in powers of x

15. $\ln(2-x)$ in powers of x

16. $\frac{1}{x}$ in powers of $x-1$

17. $\frac{1}{x^2}$ in powers of $x+2$

18. $\frac{1-x}{1+x}$ in powers of x

19. $\frac{x^3}{1-2x^2}$ in powers of x

20. $\ln x$ in powers of $x-4$

Determine the interval of convergence and the sum of each of the series in Exercises 21–26.

21. $1 - 4x + 16x^2 - 64x^3 + \cdots = \sum_{n=0}^{\infty} (-1)^n (4x)^n$

22. $3 + 4x + 5x^2 + 6x^3 + \cdots = \sum_{n=0}^{\infty} (n+3)x^n$

23. $\frac{1}{3} + \frac{x}{4} + \frac{x^2}{5} + \frac{x^3}{6} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n+3}$

24. $1 \times 3 - 2 \times 4x + 3 \times 5x^2 - 4 \times 6x^3 + \cdots$
$= \sum_{n=0}^{\infty} (-1)^n (n+1)(n+3)\, x^n$

25. $2 + 4x^2 + 6x^4 + 8x^6 + 10x^8 + \cdots = \sum_{n=0}^{\infty} 2(n+1)\, x^{2n}$

26. $1 - \frac{x^2}{2} + \frac{x^4}{3} - \frac{x^6}{4} + \frac{x^8}{5} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n+1}$

Use the technique (or the result) of Example 6 to find the sums of the numerical series in Exercises 27–32.

27. $\sum_{n=1}^{\infty} \frac{n}{3^n}$

28. $\sum_{n=0}^{\infty} \frac{n+1}{2^n}$

29. $\sum_{n=0}^{\infty} \frac{(n+1)^2}{\pi^n}$

30. $\sum_{n=1}^{\infty} \frac{(-1)^n n(n+1)}{2^n}$

31. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n 2^n}$

32. $\sum_{n=3}^{\infty} \frac{1}{n 2^n}$