



Norges teknisk-naturvitenskapelige
universitet
Department of Mathematical
Sciences

MA1102
Grunnkurs i analyse II
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Øving 10

- 1 Use Abel's theorem to compute the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}.$$

Hint: Use the power series of $\arctan(x)$ you found in exercise 2 on exercise sheet 9.

- 2 Recall that the Taylor series of a function f around a point a is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n.$$

Compute the Taylor coefficients of $\sin(x)$ and $\cos(x)$ around $x=0$ and write up the Taylor series. For which x do the Taylor series converge?

- 3 a) Let $f(x) = \sqrt{1+x}$. Show that the Taylor series can be written as

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1} (2n)!}{2^{2n} (n!)^2 (2n-1)} x^n.$$

For which x does the series converge?

- b) Use Abel's theorem and exercise (3a) to compute the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1} (2n)!}{2^{2n} n!^2 (2n-1)}.$$