Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

Øving 12

1 Let $y(x)$ be the solution of the ODE

$$y' = x + y, \qquad y(0) = 1.$$

Use the improved Euler method with step size $h = 0.1$ to approximate the values $y(0.1)$ and $y(0.2)$.

Reminder: Recall that the improved Euler method can be written as

$$y_{n+1} = y_n + \frac{h}{2}\big[f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))\big].$$

2 We study the differential nequation

$$\frac{dy}{dx} = e^{-y} - 1$$

Assume that $y(x)$ is a solution and that $y(0) = 1$. Use Eulers method with step size $h = 0,1$ to find an approximate value for $y(0,2)$.

3 Use the Weierstrass M-test to show that the following functions are continuous for all x in $\mathbb{R}$

a)

$$f(x) = \sum_{n=0}^{\infty} \frac{\cos(27^n x)}{3^n}.$$

b)

$$g(x) = \sum_{n=0}^{\infty} 3^n \sin\left(\frac{x}{27^n}\right)$$

Hint for (b): $|\sin(x)| \leq |x|$.

4 For each of the statements below determine if the statement is *true* or *false*. If the statement is falls provide a counter example or an explanation of why the statement is false. If true no proof is needed.

a) For any continuous function $f : [0, 1] \to [0, 1]$ there exists a point a i $[0, 1]$ such that $f(a) = a$.

b) For any continuous function $f : (0,1) \to (0,1)$ there exists a point a i $(0,1)$ such that $f(a) = a$.

c) The sequence $\{\frac{1}{\ln n}\}_{n=2}^{\infty}$ is Cauchy

d) The series

$$\sum_{n=2}^{\infty} \frac{1}{\ln n}$$

converges

e) The set $(0,1) \cup 2$ is compact.

f) Let $\{a_n\}$ be a bounded sequence in $\mathbb{R}$. Then there exists a convergent sub sequence of $\{a_n\}$.

g) It holds that $2 - 2i = 2\sqrt{2}e^{-i\pi/4}$.

h) For any continuous functions $f : \mathbb{R} \to \mathbb{R}$ it follows that the set

$$f^{-1}[0,1] := \{x \in \mathbb{R} : f(x) \in [0,1]\}$$

is a compact subset of $\mathbb{R}$.

Note: Only one lecture was given in MA1102 in week 14 due to easter holliday. Thus this exercise sheet contains a mix of exercises relevant for this weeks lecture and some topics covered in earlier lectures.