Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
Vår 2024

Øving 13

1 Solve the differential equations

a)
$$y'' - 4y' + 4y = 0$$

b)
$$y'' - y' - 6y = 0$$

c)
$$y'' + 2y' + 4y = 0$$

2 **a)** We are given that $y_1(x) = \sin(2x)$ and $y_2(x) = \cos(2x)$ are solutions to the homogenous differential equation

$$y'' = -4y.$$

Find the unique solution that satisfies the initial conditions $y(0) = 2, y'(\pi) = 1$.

b) We are given that $y_1(x) = e^{-\frac{1}{2}x}$ and $y_2(x) = xe^{-\frac{1}{2}x}$ are solutions to the homogenous differential equation

$$4y'' + 4y' + y = 0$$

Find the unique solution that satisfies the initial conditions $y(2) = 0, y'(2) = 2$.

3 When solving inhomogenous differential equations of the form

$$a(x)y''(x) + b(x)y'(x) + c(x)y(x) = f(x),$$

the most efficient approach may be to guess a solution. In the exercises below, you are given good guesses which share similarities with the function $f(x)$ - this is in general a good approach.

a) For which values of the constants A, B, C is the function $y(x) = Ax^2 + Bx^2 + C$ a solution to the differential equation.

$$y'' + 3y' - y = 4x.$$

b) For which values of the constants A, B is the function $y(x) = A\sin(2x) + B\cos(2x)$ a solution to the differential equation.

$$y'' + 2y' - 2y = \sin(2x).$$

c) For which values of the constants A, B is the function $y(x) = Ae^x + Bxe^x$ a solution to the differential equation.

$$y'' + 8y' - 6y = xe^x.$$

4 **a)** Solve the differential equation

$$y'' - 5y' + 6y = e^{-2x},$$

with initial conditions $y(0) = 1$ and $y'(0) = 0$.

b) Find the power series solutions $y(x) = \sum_{n=0}^{\infty} a_n x^n$ of the differential equation

$$y'' - 5y' + 6y = 0,$$

with initial conditions $y(0) = 1$ and $y'(0) = 0$.

5 **a)** Solve the differential equation

$$y'' - 2y' + y = 0$$

b) Solve the differential equation

$$y'' - 2y' + y = e^{2x},$$

with initial conditions $y(0) = 1$ and $y'(0) = 0$.

c) Find the power series solutions $y(x) = \sum_{n=0}^{\infty} a_n x^n$ of the differential equation

$$x^2 y'' + y' - 2y = 0,$$

for $x \in \mathbb{R}$.