MA1102
Grunnkurs i analyse II
Vår 2024

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Department of Mathematical Sciences

Øving 2

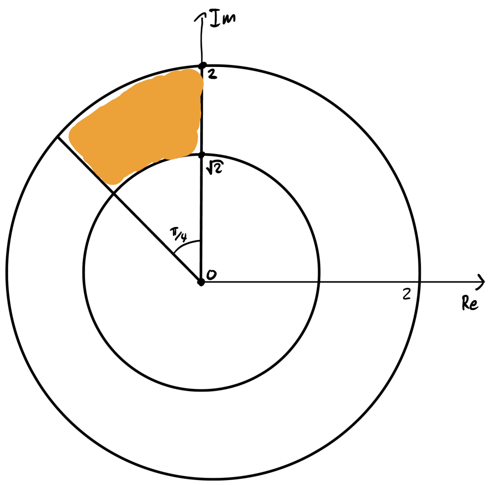
1 Find all the cube roots of the complex number $1 + i$.

2 Find all the square roots of $-1 - i$.

3 Find all complex solutions of the equation

$$z^3 - z^2 + 3z - 3 = 0.$$

4 Specify the subset of the complex plane colored orange in the following figure:



Give your answer as a set $A = \{z \in \mathbb{C} \,:\, ...\}$. Note that the picture is not to scale.

5 Let the sequence $\{a_n\}_{n\geq 1}$ be defined by $a_1 = 2$, and $a_{n+1} = 1 + 1/a_n$ for $n \geq 1$. Show that the sequence is convergent by showing that it satisfies the Cauchy criterion.

6 Show that the sequence

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \ldots$$

is bounded but does *not* satisfies the Cauchy criterion. Is the sequence convergent?

7 Let $\{a_j\}$ be a sequence of complex numbers. Suppose that for every pair of integers $N > M > 0$, it holds that

$$|a_M - a_{M+1}| + |a_{M+1} - a_{M+2}| + \cdots + |a_{N-1} - a_N| \leq 1.$$

Prove that $\{a_j\}$ converges.