Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
Vår 2024

Øving 3

1 In the lectures we have proven the following result:

Theorem 1 *Let $\{\alpha_j\}$ be a sequence of real numbers. Then the sequence is Cauchy if and only if it converges to some limit α.*

Extend this result to complex sequences. That is, prove the following:

Theorem 2 *Let $\{\alpha_j\}$ be a sequence of complex numbers. Then the sequence is Cauchy if and only if it converges to some limit α.*

2 In the lectures we have proven the following result:

Theorem 3 (Bolzano–Weierstrass) *Let $\{a_j\}$ be a bounded sequence in $\mathbb{R}$. Then there is a subsequence which converges.*

Extend this result to complex sequences. That is, prove the following:

Theorem 4 *Let $\{a_j\}$ be a bounded sequence in $\mathbb{C}$. Then there is a subsequence which converges.*

3 Let $\{\alpha_j\}$ be a convergent sequence with limit α. Prove that every subsequence converges to the limit α.

4 Let $x_1 = 2$. For $j \geq 1$, set

$$x_{j+1} = x_j - \frac{x_j^2 - 2}{2x_j}.$$

Show that the sequence $\{x_j\}$ is decreasing and bounded below. What is the limit?