Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
Vår 2024

Øving 5

Remember to justify all answers!

1 Let $\{\alpha_j\}$ be a complex sequence such that

$$\sum_{j=1}^{\infty} \alpha_j$$

converges absolutely. Show that

$$\sum_{j=1}^{\infty} \alpha_j$$

converges as well.
Hint: We have already proven this for real sequences in the lectures.

2 Determine if the following series conveges or diverges

a)

$$\sum_{n=1}^{\infty} \left(\frac{(-1)^n}{\sqrt{n}} + \frac{1}{n} \right),$$

b)

$$\sum_{n=1}^{\infty} \frac{n - \cos n}{n^2 + 2n}$$

c)

$$\sum_{n=1}^{\infty} n^2 e^{-\sqrt{n}}$$

3 For which $a \in \mathbb{R}$ is the following sum convergent

$$\sum_{n=1}^{\infty} a^n \frac{n^2}{2^n}.$$

4 (a) Let $(a_n)_{n=0}^{\infty}$ be a cauchy sequence in $\mathbb{R}$. Show that the series

$$\sum_{n=1}^{\infty}(a_n - a_{n-1})$$

is convergent.

(b) Let $(a_n)_{n=0}^{\infty}$ be a bounded sequence in $\mathbb{R}$. Show that the series

$$\sum_{n=1}^{\infty}\frac{a_n}{2^n}$$

is absolutely convergent.

5 Sums of form

$$\sum_{n=0}^{\infty}a_n(x-c)^n$$

are commonly called **power series** and notable examples include

$$\sum_{n=0}^{\infty}1\cdot(x-0)^n = \frac{1}{1-x}, \qquad \sum_{n=0}^{\infty}\frac{1}{n!}(x-0)^n = e^x$$

Note that all power series do not always converge, such as that for $\frac{1}{1-x}$ above and that the center c can be non-zero. We will learn more about these later in the course but we are already in a position to do some computations with them.

a) For which $x \in \mathbb{R}$ does the power series for $\frac{1}{1-x}$ converge? Use a convergence test.

b) For which $x \in \mathbb{R}$ does the power series for e^x converge? Use a convergence test.