Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102 Grunnkurs i analyse II Vår 2024

Øving 9

1 **a)** Express $\ln(1+x)$ as a power series around zero which converges for $|x|<1$.
Hint: Use that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ *and integrate.*

b) Express

$$\frac{1}{(1-x)^2}$$

as a power series around zero which converges for $|x|<1$.
Hint: Derivative of $\frac{1}{1-x}$.

c) Express $\ln(x^2+1)$ as a power series around zero which converges for $|x|<1$.

d) Use your answer from exercise (1c) to determine the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}(n+1)}.$$

2 **a)** Write $\frac{1}{1+x^2}$ as a power series around zero. What is the radius of convergence?

b) Write $\arctan(x)$ as a power series around zero. What is the radius of convergence?
Hint: Use exercise 2a

3 For each of the power series below find the area of convergence.

a)

$$\sum_{n=0}^{\infty} \left(\frac{nx}{2+3n}\right)^n$$

b)

$$\sum_{n=1}^{\infty} n \left(\frac{n}{2+3n}\right)^n x^{n-1}$$

c)

$$\sum_{n=1}^{\infty} \left(\frac{nx}{4n+9}\right)^{2n}$$