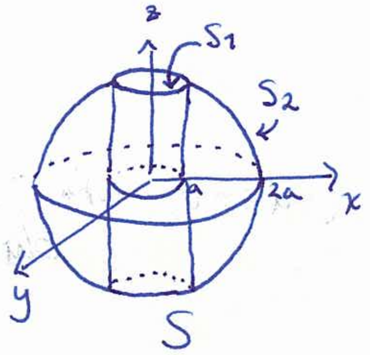
Hint oppgave 16. 4. 13

$D : x^2 + y^2 + z^2 \leq 4a^2 , \quad x^2 + y^2 \geq a^2$



a) Fluks over S ?

b) Fluks over S_1 ?

c) Fluks over S_2 ?

$S = S_1 + S_2$

Vi vet at Fluks over S er:

$$\oiint_S \mathbb{F} \cdot \hat{N} \, dS = \iint_{S_1} \mathbb{F} \cdot \hat{N} \, dS + \iint_{S_2} \mathbb{F} \cdot \hat{N} \, dS$$

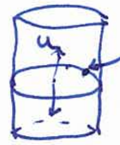
For a) S er en lukket overflate, og da kan vi bruke divergens teoremet:

$$\oiint_S F \cdot \hat{N} \, dS = \iiint_D \operatorname{div} F \, dV$$

$$= \iiint_D 3 \, dV = 3 \cdot \left(\begin{array}{c}\text{volumet} \\ \text{til } D\end{array}\right)$$

Hint: bruk sylinderkoordinater for å beregne $\iiint_D dV$.

b) Her kan vi IKKE bruke divergens teoremet fordi S_1 er ikke lukket!

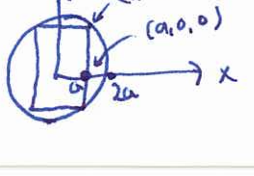


$(a\cos v, a\sin v, u)$

$\rightsquigarrow$ Vi må parametrisere S_1:
- parametriserer sirkelen av radius a i xy-planet
- høyden er bare et annet parameter u.

$r(u,v) = (a\cos v, a\sin v, u) \qquad 0 \leq v \leq 2\pi$

Finner grensene til $u \rightsquigarrow$ ser på en skive av kulen i xz planet



$x = a$
$y = 0$
$z = ?$

bruk $x^2 + y^2 + z^2 = 4a^2$

$\rightsquigarrow a^2 + 0^2 + z^2 = 4a^2 \implies z = \pm\sqrt{3}a$

så $\quad -\sqrt{3}\,a \le u \le \sqrt{3}\,a$

$$N\,dS = \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right) du\,dv$$

$$= (-a\cos v, -a\sin v, 0)\, du\,dv$$

så $$\iint_{S_1} \mathbb{F}\cdot N\,dS = \int_{-\sqrt{3}a}^{\sqrt{3}a} \int_0^{2\pi} \mathbb{F}(r(u,v)) \cdot (-a\cos v, -a\sin v, 0) \cdot dv\,du$$

$$= \int_{-\sqrt{3}a}^{\sqrt{3}a} \int_0^{2\pi} -a^2\,dv\,du = \underline{\underline{-4\sqrt{3}\,a^3\pi}}$$

c) $$\iint_{S_2} \mathbb{F}\cdot N\,dS = \oiint_S \mathbb{F}\cdot N\,dS - \iint_{S_1} \mathbb{F}\cdot N\,dS$$

$$= 12\sqrt{3}\,\pi a^3 - (-4\sqrt{3}\,\pi a^3)$$

$$= \underline{\underline{16\sqrt{3}\,\pi a^3}}$$