LØSNINGSSKISSER MA1103

Vanlige forbehold! KH.

<u>Oppgave 1</u> f er 0 på aksene slik at $f_x(0,0) = 0 = f_y(0,0)$. f er ikke deriverbar i $(0,0)$ da f ikke engang er kontinuerlig i $(0,0)$.

<u>Oppgave 2</u> $x = r\cos\theta$, $y = r\sin\theta$. Altså er

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \cos\theta\frac{\partial z}{\partial x} + \sin\theta\frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = -r\sin\theta\frac{\partial z}{\partial x} + r\cos\theta\frac{\partial z}{\partial y}$$

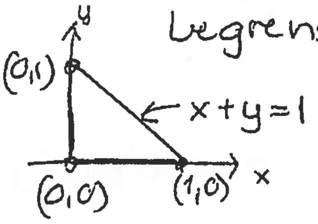
slik at

$$\underline{\left(\frac{\partial z}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial z}{\partial \theta}\right)^2} = \left(\cos\theta\frac{\partial z}{\partial x} + \sin\theta\frac{\partial z}{\partial y}\right)^2 + \left(-\sin\theta\frac{\partial z}{\partial x} + \cos\theta\frac{\partial z}{\partial y}\right)^2$$

$$= (\cos^2\theta + \sin^2\theta)\left(\frac{\partial z}{\partial x}\right)^2 + (\sin^2\theta + \cos^2\theta)\left(\frac{\partial z}{\partial y}\right)^2$$

$$= \underline{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}.$$

<u>Oppgave 3</u> En kont. funksjon på et lukket begrenset område har såvel maks. som min. Oppnås på randa eller i indre punkt der $f_x = f_y = 0$.



Da $f(x,y) = 27xy(1-(x+y))$, er <u>f lik 0 på randa.</u>

Kritiske punkt i det indre:

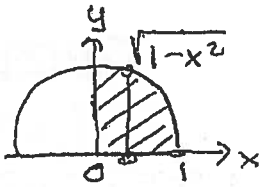
$$\left.\begin{array}{l} f_x = 27y(1-x-y) - 27xy = 27y(1-2x-y) = 0 \\ f_y = 27x(1-2y-x) = 0 \end{array}\right\}$$

<u>Indre kritisk punkt:</u> $\left.\begin{array}{l} y + 2x = 1 \\ x + 2y = 1 \end{array}\right\} \Leftrightarrow \underline{x = y = \frac{1}{3}}$

Maksimum: $f(\frac{1}{3}, \frac{1}{3}) = \underline{\underline{1}}$ Minimum: $\underline{\underline{0}}$

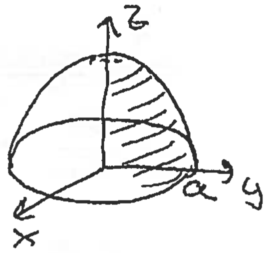
Oppgave 4

$$\int_0^1\int_0^{\sqrt{1-x^2}}(x^2+y^2)\,dy\,dx = \int_0^{\frac{\pi}{2}}\int_0^1 r^2\,r\,dr\,d\theta = \underline{\underline{\frac{\pi}{8}}}$$



Oppgave 5

Massen er gitt ved



$$k\iiint_{H(\text{halvkule})}\delta(x,y,z)\,dV = k\iiint_H (x^2+y^2+z^2)\,dV$$

$$= k\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^a \rho^2\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = k\,2\pi\,\frac{a^5}{5}\left[-\cos\phi\right]_0^{\frac{\pi}{2}} = \underline{\underline{\frac{2k\pi}{5}a^5}}$$

Oppgave 6

$$\omega\oint_C -y\,dx + x\,dy \underset{\text{Green}}{=} \omega\iint_R 2\,dA = \underline{\underline{2\pi a^2\omega}}$$

Har antatt C positivt orientert; i motsatt fall blir svaret $-2\pi a^2$.

<u>Alternativt</u> $x-x_0 = a\cos t$, $y-y_0 = a\sin t$; $0\le t\le 2\pi$,

slik at $\omega\oint_C -y\,dx + x\,dy$

$$= \omega\int_0^{2\pi}\left[\pm(y_0+a\sin t)\sin t + (x_0+a\cos t)\cos t\right]dt$$

$$= \underline{\underline{2\pi\omega a^2}}$$

<u>Bemerker.</u> Om du legger sentrum (x_0,y_0) til $(0,0)$, vil jeg trekke lite/ingenting!

Oppgave 7

Søker $f(x)$ slik at $\operatorname{curl} \mathbb{F} = \mathbb{0}$:

$$\operatorname{curl} \mathbb{F} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2 z & yf(x) \end{vmatrix} = \langle f(x)-x^2, 2xy - yf'(x), 0 \rangle = \mathbb{0}$$

$\Leftrightarrow$ $\underline{\underline{f(x) = x^2}}$

Da feltet $\mathbb{F} = \langle 2xyz, x^2 z, x^2 y \rangle$ er definert i hele $\mathbb{R}^3$ (og glatt), er linjeintegralet uavhengig av vegen; feltet er konservativt.

Oppgave 8

$$\mathbb{F} = \left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$$

a) $$\frac{\partial F_1}{\partial x} = \frac{(x^2+y^2+z^2)^{3/2} - \frac{3}{\cancel{2}}(x^2+y^2+z^2)^{1/2} \cdot \cancel{2}x^2}{(x^2+y^2+z^2)^3}$$

Vi får tilsvarende uttrykk for $\frac{\partial F_2}{\partial y}$ og $\frac{\partial F_3}{\partial z}$, og tilsammen

$$\nabla \cdot \mathbb{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = \frac{3(x^2+y^2+z^2)^{3/2} - 3(x^2+y^2+z^2)^{1/2}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$$

$$= \underline{\underline{0}}$$

b) $$\iint_S \mathbb{F} \cdot \hat{\mathbb{N}}\, dS = \iint_S \frac{\mathbb{r}}{|\mathbb{r}|^3} \cdot \frac{\mathbb{r}}{|\mathbb{r}|}\, dS = \iint_S \frac{1}{a^2}\, dS = \underline{4\pi} \neq 0$$

Dette er ikke i strid med divergensteoremet da $\mathbb{F}$ ikke er definert (enn si kontinuerlig) i $(0,0,0)$ som omsluttes av S.

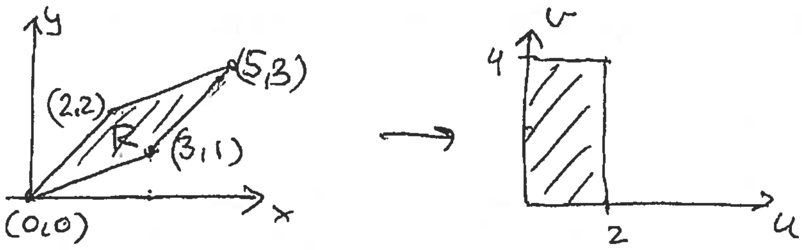
Oppgave 9

Vi innfører $u = x - y$, $v = 3y - x$ og har

$$\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left|\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix}\right| = \left|\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}\right| = 2,$$ slik at

$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2}$. Videre har vi $(x,y) \to (u,v)$, dvs.

$(0,0) \to (0,0)$, $(3,1) \to (2,0)$, $(5,3) \to (2,4)$, $(2,2) \to (0,4)$.



$$\int_R (x-y)^m (3y-x)^n \, dx\,dy = \frac{1}{2}\int_0^4 \int_0^2 u^m \cdot v^n \, du\,dv$$

$$= \underline{\underline{\frac{2^m \cdot 4^{n+1}}{(m+1)(n+1)}}}$$

$$\left(= \underline{\underline{\frac{2^{m+2n+2}}{(m+1)(n+1)}}} \right)$$