

$$2x^2 - 4xy - y^2 - 4x - 8y = -14$$

Opg. 9.6.11

$$\begin{bmatrix} x & y \end{bmatrix} \underbrace{\begin{bmatrix} 2 & -2 \\ -2 & -1 \end{bmatrix}}_A \begin{bmatrix} x \\ y \end{bmatrix} + \underbrace{\begin{bmatrix} -4 & -8 \end{bmatrix}}_B \begin{bmatrix} x \\ y \end{bmatrix} = -14 \quad (*)$$

$$\lambda_1 = 3: \text{eigenwaarde } z \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} \text{ velg } \underline{v} = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$$\lambda_2 = -2: \text{eigenwaarde } s \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ velg } \underline{u} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$P = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix} \rightsquigarrow P^T A P = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$$

Koördinaatsstrijke: $\underline{x} = P \underline{x}' \iff \begin{bmatrix} x \\ y \end{bmatrix} = P \begin{bmatrix} x' \\ y' \end{bmatrix}$

$$(*) \rightsquigarrow \underline{x}'^T A \underline{x}' + B \underline{x}' = -14$$

$$(\underline{P} \underline{x}')^T A (\underline{P} \underline{x}') + B (\underline{P} \underline{x}') = -14$$

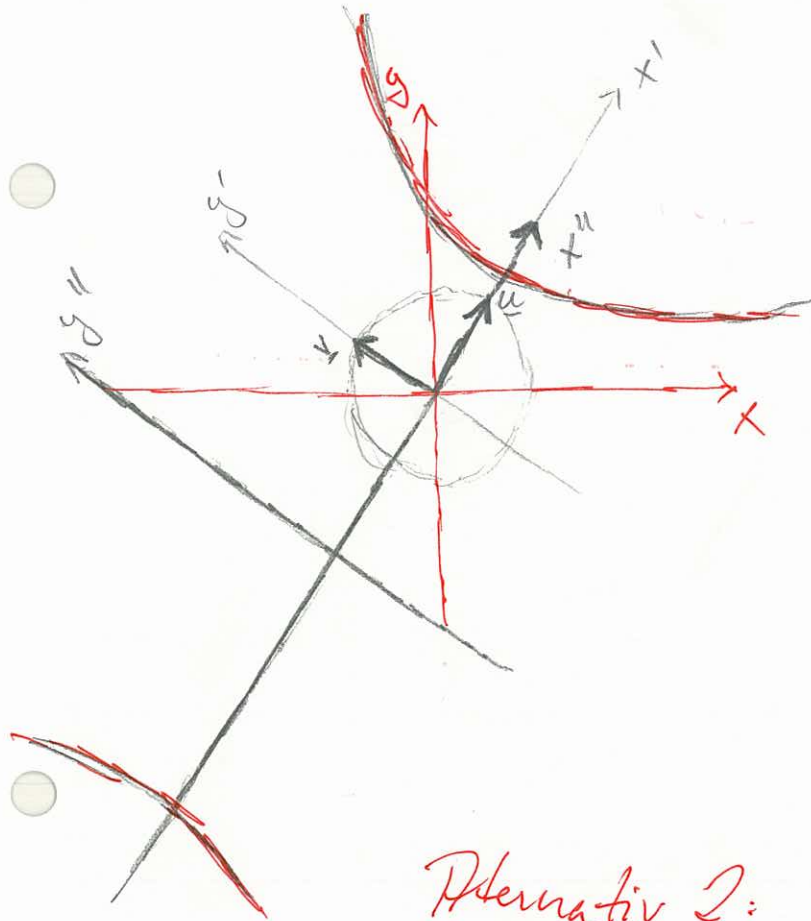
$$\underline{x}'^T P^T A P \underline{x}' + B \underline{P} \underline{x}' = -14$$

$$\begin{bmatrix} x' & y' \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} -4 & -8 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = -14$$

$$-2x'^2 + 3y'^2 - 4\sqrt{5}x' = -14$$

$$-2(x' + \sqrt{5})^2 + 3y'^2 = -24 \quad (2(x' + \sqrt{5})^2 - 3y'^2 = 24)$$

$$\frac{x''^2}{(2\sqrt{3})^2} - \frac{y''^2}{(\sqrt{2})^2} = 1$$

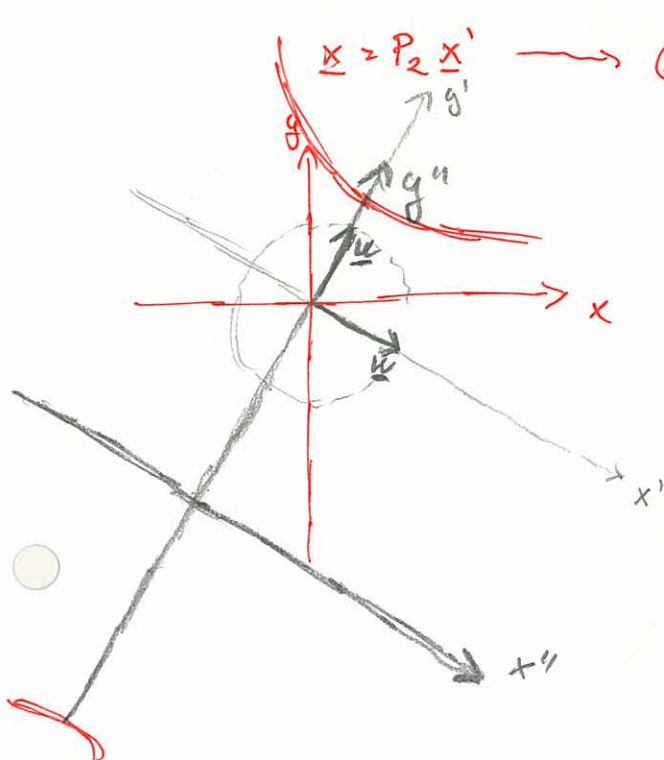


Alternativ 2:

Velg $\underline{w} = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$ i stedet for \underline{v} , og la

$$P_2 = [\underline{w} \quad \underline{u}] = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \rightsquigarrow P_2^T A P_2 = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

Koordinatshifte: (Merke x' her ikke sammen som x' over, osv.)



$$\underline{x} = P_2 \underline{x}' \rightsquigarrow (*) \rightarrow \underline{x}'^T \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \underline{x}' + [-4 \quad -8] \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \underline{x}' = -14$$

$$3x'^2 - 2y'^2 - 2\sqrt{5}y' = -14$$

$$3x'^2 - 2(y' + \sqrt{5})^2 = -24$$

$$\frac{y''^2}{(2\sqrt{2})^2} - \frac{x''^2}{(2\sqrt{2})^2} = 1$$