Norges teknisk-
naturvitenskapelige universitet
Institutt for matematiske fag

CANDIDATE NR.:

MIDTERM EXAM IN MA1201 LINEAR ALGEBRA AND GEOMETRY, FALL 2009

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Time: 12.15 - 13.45
English

Permited aids: No printed or written aides allowed. Permitted calulator is HP30S or CITIZEN SR-270X.

The midterm exam has **6** double sided pages and consists of two parts: Problems 1 to 4 are multiple choice. In this part every problem has 5 alternatives and only one of these is correct. Check off the correct alternative. In problem 5 to 9 you must give arguments for all your answers. Each problem counts equally. Write your answer directly on these pages. You must write your candidate number on **each** sheet.

Good luck!

Problem 1

Supose that A is a 2×2-matrix and that A^{-1} is given by:

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}.$$

Then A is equal to

$\boxed{\text{A}}$ $\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$, $\boxed{\text{B}}$ $\frac{1}{13}\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$, $\boxed{\text{C}}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$\boxed{\text{D}}$ $\frac{1}{13}\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$, $\boxed{\text{E}}$ $\frac{1}{13}\begin{bmatrix} -5 & -1 \\ 3 & -2 \end{bmatrix}$

Problem 2

Given the system of linear equations:

$$\begin{array}{rcrcrcr} 3x & + & 2y & - & z & = & -15 \\ 5x & + & 3y & + & 2z & = & 0 \\ 3x & + & y & + & 3z & = & 11 \\ -6x & - & 4y & + & 2z & = & 30 \end{array}$$

The reduced row-echolon form of the augmented matrix is:

$\boxed{\text{A}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{array}\right]$, $\boxed{\text{B}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{array}\right]$, $\boxed{\text{C}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$,

$\boxed{\text{D}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 7 & 45 \\ 0 & 1 & -11 & 40 \\ 0 & 0 & 1 & 7 \end{array}\right]$, $\boxed{\text{E}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 3 \end{array}\right]$

Problem 3

Let A, B and C be $n \times n$-matrices. Then one of the following is valid:

$\boxed{\text{A}}$ If $AB = AC$, then $B = C$.

$\boxed{\text{B}}$ If A is invertible, then $AB = AC$.

$\boxed{\text{C}}$ If A is invertible and $BA = CA$, then $B = C$.

$\boxed{\text{D}}$ For each pair of $n \times n$-matrices A and B we have that $(AB)^T = A^T B^T$.

$\boxed{\text{E}}$ If $\det(A) = 0$, then A is invertible.

Problem 4

If the matrix

$$A = \begin{bmatrix} 0 & 3 & 0 & 1 \\ 1 & 0 & -2 & 2 \\ 1 & 3 & -2 & 3 \\ -5 & 5 & 1 & -1 \end{bmatrix},$$

Then $\det(A)$ is equal to:

$\boxed{\text{A}}$ 0, $\boxed{\text{B}}$ 27, $\boxed{\text{C}}$ -13, $\boxed{\text{D}}$ 2, $\boxed{\text{E}}$ 161.

Problem 5

Solve the system of linear equations:

$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 3 \\ x & - & y & & & = & 0 \\ 2x & & & + & z & = & 3 \end{array}$$

How many solutions does it have?

Problem 6

a) Determine A^{-1} if $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & -1 \end{bmatrix}$.

b) Use that $\mathbf{x} = A^{-1}\mathbf{b}$ to solve the system of linear equations

$$\begin{array}{rcrcrcl} x & + & 2y & & & = & 4 \\ & & y & + & 3z & = & 1 \\ 2x & & & - & z & = & 0 \end{array}$$

c) Use Cramer's rule to solve the system of linear equations given in **b)**.

Problem 7

Show that

$$\det\left(\begin{bmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\right) = 0$$

is the equation for a straight line in the plane $\mathbb{R}^2$, and that the points $(1, 2)$ and $(0, 1)$ are incident with it.

Problem 8

a) Show that if $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, then $A^2 = 0$ (the zero-matrix).

b) Compute A^2 and A^3 if $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

c) Let A be a $n \times n$ matrix such that $A^k = 0$ for a natural number k. Show that A is not invertible.

d) Let A be a square matrix such that $A^4 = 0$. Show that $(I - A)^{-1} = I + A + A^2 + A^3$, where I is the identity matrix.

Oppgave 9

For which values of α will the following system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{array}{rcrrcr} 3x & - & 2y & +z & = & 1 \\ -x & + & y & -\alpha z & = & 0 \\ 2x & - & y & +(\alpha-1)^2 z & = & \alpha \end{array}$$