NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **MA1201/MA6201 Lineær algebra**

Academic contact during examination: Steffen Oppermann

Phone: 91897712

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Permitted examination support material: No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

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Number of pages: 3

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Checked by:

Date Signature

Problem 1

a) Find all solutions to the system of linear equations

$$\begin{aligned} x-2y \quad &= -3 \\ x+\ y-\ z &= \ \ 2 \\ 2x-\ y+\ z &= \ \ 1 \\ 2x+\ y+2z &= \ \ 6 \end{aligned}$$

b) Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

Is A invertible? (Give a reason for your answer.)

Problem 2 Consider the two vectors $\vec{u} = \left[\begin{smallmatrix} 2 \\ 1 \\ 2 \end{smallmatrix}\right]$ and $\vec{v} = \left[\begin{smallmatrix} 1 \\ 0 \\ -1 \end{smallmatrix}\right]$ in $\mathbb{R}^3$.

a) Show that $\vec{u}$ and $\vec{v}$ are orthogonal. Normalize them to obtain a pair of orthonormal vectors $\vec{u}_n$ and $\vec{v}_n$.

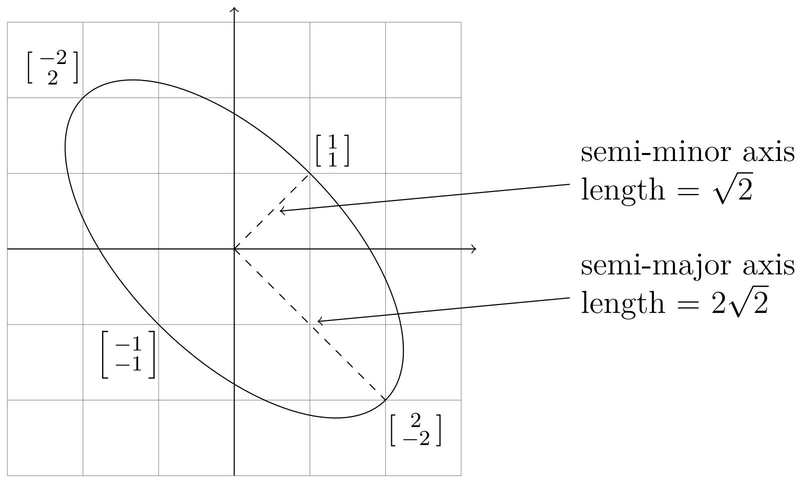
b) Find a third vector $\vec{w}$ such that $(\vec{u}_n, \vec{v}_n, \vec{w})$ is an orthonormal basis of $\mathbb{R}^3$.

(Hint: For $\mathbb{R}^3$ there is a construction which “creates” a vector orthogonal to two given ones.)

c) Show in general: two orthogonal non-zero vectors in $\mathbb{R}^n$ are linearly independent.

(Remember: A collection of vectors is called *orthogonal* if the dot product of any two distinct vectors from the collection is zero. It is called *orthonormal* if, in addition, all the vectors have length 1.)

Problem 3 Consider the ellipse



Find the equation descibing this ellipse.

(Hint: A strategy might be to first find the equation in a coordinate system that is rotated by a certain angle.)

Problem 4 Let A be a square matrix such that $A^2 + A + I = 0$.

a) Show that A does not have any real eigenvalues.

b) Which two complex numbers can be eigenvalues of A? Write them in polar form.

c) Find a real 2×2-matrix A that satisfies the equation above.

Problem 5 Let A be an $m \times n$-matrix and B be an $n \times m$-matrix, such that $AB = I_m$. Decide if the following claims are true. If they are, give a proof. If not, give a counter example.

a) If C is an $n \times m$-matrix such that $CA = I_n$, then $C = B$.

b) If C is an $n \times m$-matrix such that $AC = I_m$, then $C = B$.

Problem 6 Let A be a real 2×2-matrix with (at least one) real eigenvalue.

Assume there is a real 2×2-matrix B having no real eigenvalues such that $AB = BA$.

Show that $A = \left[\begin{smallmatrix} a & 0 \\ 0 & a \end{smallmatrix}\right]$ for some $a \in \mathbb{R}$.

(Hint: What can be said about complex eigenvalues and eigenvectors of B? Can something be said about the product of A with these complex vectors?)