

## 11.11 Datagrafikk

Ulike transformasjoner:



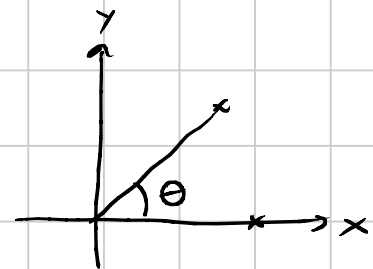
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$$\text{Skalering: } \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

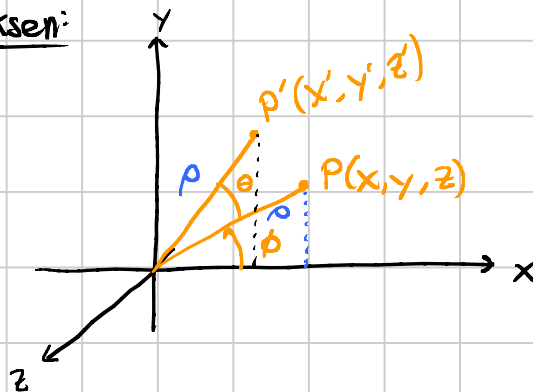
$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

Rotasjon:

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



Rotasjon om z-aksen:



$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$x' = \rho \cdot \cos(\phi + \theta) = \overbrace{\rho \cos \phi}^x \cos \theta - \overbrace{\rho \sin \phi}^y \sin \theta \\ = x \cos \theta - y \sin \theta$$

$$y' = \rho \sin(\phi + \theta) = \overbrace{\rho \cos \phi}^x \sin \theta + \overbrace{\rho \sin \phi}^y \cos \theta \\ = x \sin \theta + y \cos \theta$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



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Rotasjon om x-aksen:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

Rotasjon om y-aksen:

$$\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

### Oppgave 11.11.5

a) 1. Skaler med 0.3 i x-retning og 0.5 i y-retning

$$M_1 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Roter 45° om x-aksen

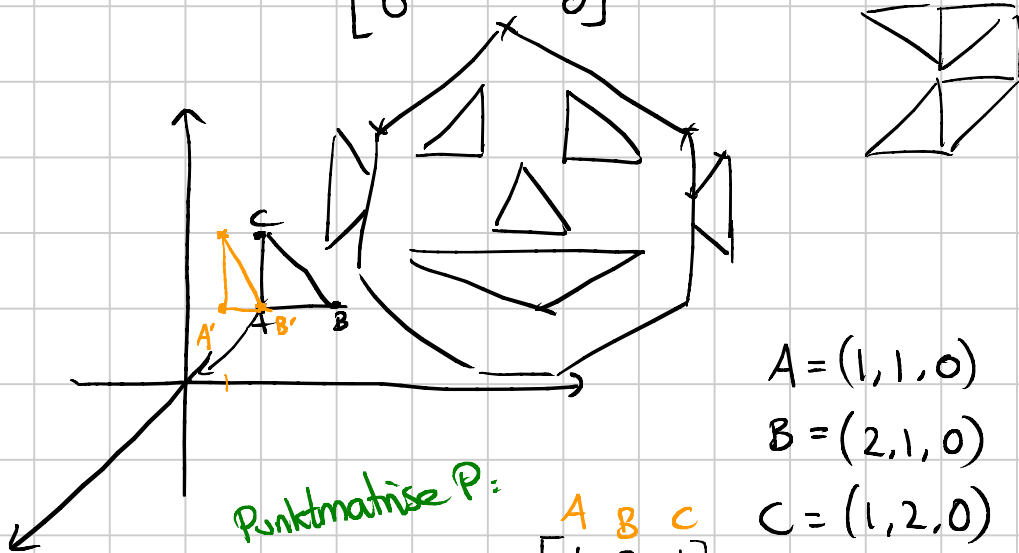
$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

3. Transler 1 enhet i x-retningen [ 2D:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & & & \\ 0 & \dots & & \\ 0 & & & 1 \\ & & & & 0 \end{bmatrix}$$



$$A = (1, 1, 0)$$

$$B = (2, 1, 0)$$

$$C = (1, 2, 0)$$

$$A \leftrightarrow B$$

$$B \rightarrow C$$

$$C \rightarrow A$$

Punktmatrix P:

$$P = \begin{matrix} & A & B & C \\ \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

A' B' C'

Ved f.eks. skalering må vi

1) translere til origo,

2) skalere

3) translere tilbake

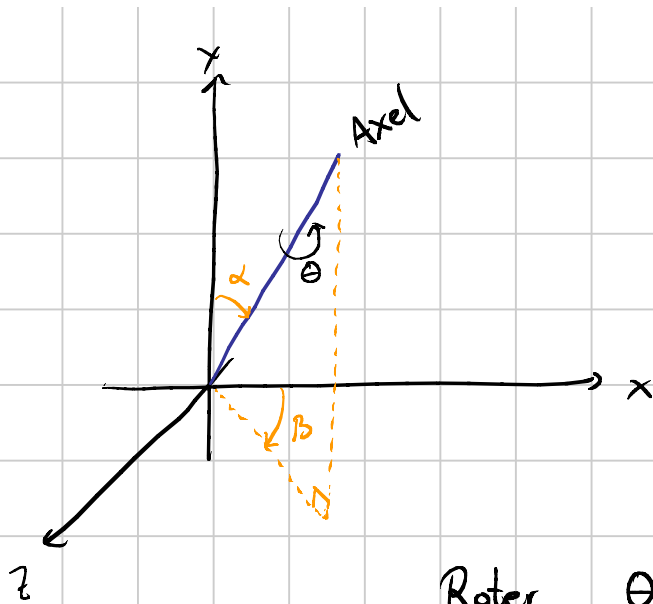
$$P' = T_a P$$

$$P'' = S P'$$

$$P''' = T_{-a} P''$$



$$P''' = \overset{3}{T_{-a}} \overset{2}{S} \overset{1}{T_a} P$$



$$R_{x,0}$$

$$R_{y,0}$$

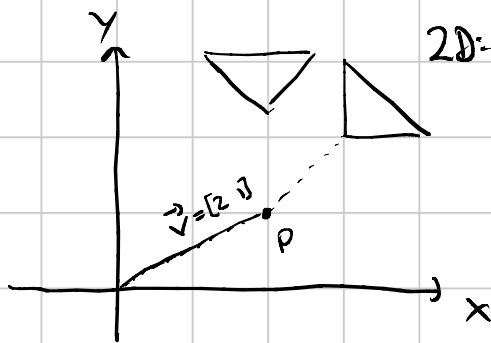
$$R_{z,0}$$

$$R_{y,-\beta}$$

Roter  $\theta$  om Axel

$$R = R_{y,-\beta} R_{z,-\alpha} R_{y,0} R_{z,\alpha} R_{y,\beta}$$

$$T_y R T_y^{-1}$$



enstet ä rotere  $45^\circ$  om P.

$$M = T_y R_\theta T_y^{-1}$$

$$T_y^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \underline{\underline{\quad}} \underline{\underline{\quad}}$$