## 8.4 Matriser til lin. trans.

Anta $T: U \to V$ er en lineær transformasjon og $\dim(U) = n$, $\dim(V) = m$ og

$B = \{u_1, \ldots, u_n\}$ en basis for $U$ og

$B' = \{v_1, \ldots, v_m\}$ en basis for $V$.

Vi skal finne en $A$ $m \times n$ s.a. $\forall x \in U$ har vi $A[x]_B = [T(x)]_{B'}$

$U$ $\qquad\qquad$ $V$

$$\begin{tikzcd} x \arrow[r, "T"] \arrow[d] & T(x) \arrow[d] \\ {[x]_B} \arrow[r, "A"] & {[T(x)]_{B'}} \end{tikzcd}$$

$\mathbb{R}^n$ $\qquad\qquad$ $\mathbb{R}^m$

**On the fly ex.**

La $T: P_2 \to P_1$

$T(p(x)) = p'(x)$

$B = \{1, x, x^2\}$, $B' = \{1, x\}$

$p(x) = 1 + 3x^2$

$T(p(x)) = 6x$

$[p(x)]_B = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = 1 \cdot 1 + 3 \cdot x^2$

$[T(p(x))]_{B'} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$$A[x]_B = [T(x)]_{B'} \qquad \in U$$

$$A[u_1]_B = [T(u_1)]_{B'} \qquad [u_1]_B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \{u_1, \ldots, u_n\}$$

A's 1. kolonne

Dermed er $A = \left[\; \underline{[T(u_1)]_{B'}} \;\; \underline{[T(u_2)]_{B'}} \;\; \ldots \;\; \underline{[T(u_n)]_{B'}} \;\right]$

HØGSKOLEN I SØR-TRØNDELAG

$T(1) = 0 \qquad [T(1)]_{B'} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$T(x) = 1 \qquad [T(x)]_{B'} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T(x^2) = 2x \qquad [T(x^2)]_{B'} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Sjekk: hva er $T(1+3x^2)$?

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ 6 \end{bmatrix}}} \leftarrow [T(1+3x^2)]_{B'}$$

$$0 \cdot v_1 + 6 \cdot v_2 = \underline{\underline{6 \cdot x}}$$

$$A = [T]_{B',B}$$

For lineære operatorer: $T: V \to V$, $B = \{u_1, \dots, u_n\}$

$$[T]_B = \big[\,[T(u_1)]_B \;\dots\; [T(u_n)]_B\,\big] \quad \text{og er s.a.}$$

$$[T(x)]_B = [T]_B\,[x]_B$$

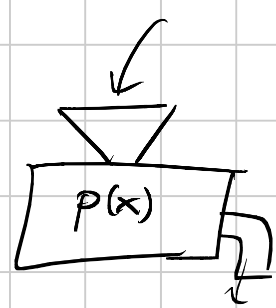
$p(x) \in P_2$

**Eksempel:** $T: P_2 \to P_2$, $B = \{1, x, x^2\}$

$T(p(x)) = p(2x+1)$

$T(x^2) = (2x+1)^2 = 4x^2 + 4x + 1$

$T(x) = 2x+1$



$$T(1) = 1$$

$$[T(x^2)]_B = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \qquad [T(x)]_B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad [T(1)]_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[T]_B = \big[\, [T(1)]_B \;\; [T(x)]_B \;\; [T(x^2)]_B \,\big] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix}$$

Hva er $T(\underline{1+x^2})$ ?

$$T(p(x)) = 1 + (2x+1)^2 = 4x^2+4x+2$$

$$\downarrow$$

$$\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$\begin{tikzcd} x \arrow[r, "\text{direkte}"] \arrow[d] & T(x) \\ {[x]_B} \arrow[r] & {[T(x)]_B} \arrow[u] \end{tikzcd}$$

$B = \{1, x, x^2\}$

$$[p(x)]_B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$[T(x)]_B = [T]_B [x]_B$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

Finn $T^{-1}$ og verifiser at $[T^{-1}]_B = [T]_B^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

Anta $\quad q(x) = T(p(x)) = p(2x+1)$

Vi vil finne $\quad T^{-1}(q(x)) = p(x)$

Hvis $\quad q(x) = a_2x^2 + a_1x + a_0 \qquad p(x) = b_2x^2 + b_1x + b_0$ , da

må $\quad a_2x^2 + a_1x + a_0 = b_2(2x+1)^2 + b_1(2x+1) + b_0$

Som gir

$$a_2 = 4b_2$$
$$a_1 = 4b_2 + 2b_1$$
$$a_0 = b_2 + b_1 + b_0$$

løser for $b_0, b_1, b_2$

$$b_2 = \frac{a_2}{4}$$
$$b_1 = \frac{a_1 - a_2}{4}$$
$$b_0 = a_0 + \frac{a_2 - 2a_1}{4}$$

Dermed er $T^{-1}(q(x)) = \frac{a_2}{4}x^2 + \frac{a_1 - a_2}{2}x + \left(a_0 + \frac{a_2 - 2a_1}{4}\right) = p(x)$

$$[T^{-1}]_B = \left[ [T^{-1}(1)]_B \ [T^{-1}(x)]_B \ [T^{-1}(x^2)]_B \right]$$

$T^{-1}(1) = 1 \quad (a_0 = 1) \qquad [T^{-1}(1)]_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$T^{-1}(x) = \frac{1}{2}x - \frac{1}{2} \quad (a_1 = 1) \qquad [T^{-1}(x)]_B = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix}$

$T^{-1}(x^2) = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4} \quad (a_2 = 1)$

$[T^{-1}(x^2)]_B = \begin{bmatrix} 1/4 \\ -1/2 \\ 1/4 \end{bmatrix}$

Så $[T^{-1}]_B = \begin{bmatrix} 1 & -1/2 & 1/4 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1/4 \end{bmatrix} = [T]_B^{-1}$