Midt semester 2007
MA1202

Opg 1

$$\begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & -2 & -4 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank $A = 3$

nullity $A = 2$

Opg 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & a^2 \\ 2 & 1 & 3 & -3a \\ 3 & 1 & 4 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & a^2 \\ 0 & 1 & 1 & -3a-2a^2 \\ 0 & 1 & 1 & -2-3a^2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & a^2 \\ 0 & 1 & 1 & -3a-2a^2 \\ 0 & 0 & 0 & -2+3a-a^2 \end{array}\right]$$

$$-2+3a-a^2 = 0$$

$$a = \frac{-3 \pm \sqrt{9-8}}{-2} = \frac{-3 \pm 1}{-2} = \begin{cases} 1 \\ 2 \end{cases}$$

$a=1$:
$$x_1 + x_3 = 1$$
$$x_2 + x_3 = -5$$
sett $x_3 = 0$

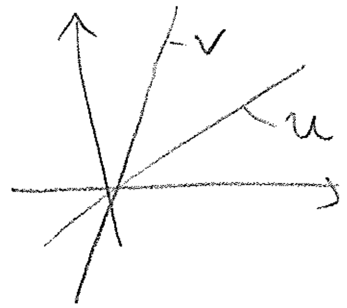
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - 5 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} a^2 \\ -3a \\ -2 \end{pmatrix}$$

$a=2$:
$$x_1 + x_3 = 4$$
$$x_2 + x_3 = -14$$

$$4 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - 14 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ -2 \end{pmatrix} = \begin{pmatrix} a^2 \\ -3a \\ -2 \end{pmatrix}$$

Opg 3

La



$U \cup V$ er ikke lukka under addisjon $\lightning$

$U \setminus V$ & $V \setminus U$ inneholder ikke null-elementet $\lightning$

$U \cap V$:

La $a, b \in U \cap V$, r skalar

Da er $ra, a+b \in U$ siden U er et underrom

& $ra, a+b \in V$ siden V er et underrom.

$\Rightarrow ra, a+b \in U \cap V$

Så $U \cap V$ er et underrom av V.

Opg 4

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$-\frac{1}{2}(t+1) - \frac{1}{2}(t-1) + 1(t+t^2)$$

$$= -\frac{1}{2}t - \frac{1}{2} - \frac{1}{2}t + \frac{1}{2} + t + t^2 = t^2$$

$$\Rightarrow (t^2)_S = \left(-\frac{1}{2}, -\frac{1}{2}, 1\right)$$

Ops 5

$$d(\underline{u}, 2\underline{v}) = \| \underline{u} - 2\underline{v} \|$$

$$\| \underline{u} - 2\underline{v} \|^2 = \langle \underline{u} - 2\underline{v} , \underline{u} - 2\underline{v} \rangle$$

$$= \langle \underline{u}, \underline{u} \rangle - 4 \langle \underline{u}, \underline{v} \rangle + 4 \langle \underline{v}, \underline{v} \rangle$$

$$= \| \underline{u} \|^2 - 4 \langle \underline{u}, \underline{v} \rangle + 4 \| \underline{v} \|^2$$

$$= 3 - 4 \cdot 2 + 4 \cdot 2 = 3$$

$$\Rightarrow \quad d(\underline{u}, 2\underline{v}) = \sqrt{3}$$

Ops 6

$$\dim W = 2 \quad \Rightarrow \quad \dim W^\perp = 1$$

$$(1,1,1) \cdot (1, \tfrac{1}{2}, \tfrac{1}{2}) = 1 + \tfrac{1}{2} + \tfrac{1}{2} = 2 \neq 0$$

$$\Rightarrow \quad W^\perp \neq \operatorname{Span} \{ (1, \tfrac{1}{2}, \tfrac{1}{2}) , (-2,-1,-1) \}$$

$$(1, -\tfrac{1}{2}, -\tfrac{1}{2}) \cdot (a(0,-1,1) + b(1,1,1))$$

$$= a \, (1, -\tfrac{1}{2}, -\tfrac{1}{2}) \cdot (0,-1,1) + b \, (1, -\tfrac{1}{2}, -\tfrac{1}{2}) \cdot (1,1,1)$$

$$= a \left(\tfrac{1}{2} - \tfrac{1}{2} \right) + b \left(1 - \tfrac{1}{2} - \tfrac{1}{2} \right) = 0$$

$$\Rightarrow \quad \operatorname{Span} \{ (1, -\tfrac{1}{2}, -\tfrac{1}{2}) \} \subset W^\perp$$

Siden de har samme dimensjon er

$$W^\perp = \operatorname{Span} \{ (1, -\tfrac{1}{2}, -\tfrac{1}{2}) \}$$

Opg 7

$$P - I = \frac{1}{10}\begin{bmatrix} -9 & 1 & 7 \\ 2 & -2 & 2 \\ 7 & 1 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & -8 & 16 \\ 0 & 8 & -16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_1 - x_3 &= 0 \\ x_2 - 2x_3 &= 0 \end{aligned}$$

$$q = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad 1 + 2 + 1 = 4$$

$\Rightarrow \frac{1}{4}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ er den stabile sannsynlighetsv.

Opg 8

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & a \\ 2 & 1 & 1 & 2 & b \\ 1 & -7 & 2 & -2 & c \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & a \\ 0 & 5 & -1 & 2 & b-2a \\ 0 & -5 & 1 & -2 & c-a \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & a \\ 0 & 5 & -1 & 2 & b-2a \\ 0 & 0 & 0 & 0 & c-a+b-2a \end{array}\right]$$

$\Rightarrow -3a + b + c = 0$ & $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -7 \end{bmatrix} \right\}$ er en basis for kolonnerommet

$$\Rightarrow \quad L = \{ (a, b, c) \mid 3a = b + c \}$$

$$= \text{Span}\{ (1,2,1), (-2,1,-7) \}$$