Løsningsforslag Øving 6
MA1202/6202 V2016

Kap6

Oppg 8

a)
$$\langle 3 - x + 2x^2, 2 - 4x^2 \rangle$$
$$= 3\cdot 2 + (-1)\cdot 0 + 2\cdot(-4) = -2$$

b)
$$\langle -5 + 2x + x^2, 3 + 2x - 4x^2 \rangle$$
$$= (-5)\cdot 3 + 2\cdot 2 + 1\cdot(-4) = -15$$

Oppg 16

Her er
$$\langle p, q \rangle = p(-2)q(-2) + p(0)q(0) + p(2)q(2)$$
Så med $p(x) = 1 - 2x + 3x^2$ $q(x) = 3 + x^2$

har vi

x	-2	0	2
$p(x)$	17	1	9
$q(x)$	7	3	7
$p(x)-q(x)$	10	-2	2

a) Så $\langle p, q \rangle = 17\cdot 7 + 1\cdot 3 + 9\cdot 7 = 185$

b) $\|p\| = (17^2 + 1^2 + 9^2)^{\frac{1}{2}} = \sqrt{371}$

c) $d(p,q) = \|p - q\| = (10^2 + (-2)^2 + 2^2)^{\frac{1}{2}} = \sqrt{108}$

Oppg 20

a) $\langle u+v, v+w\rangle = \langle u,v\rangle + \langle u,w\rangle$
$+\langle v,v\rangle + \langle v,w\rangle$

$= 2 + 5 + 2^2 - 3 = 8$

f) $\| u - 2v + 4w \|^2$

$= \langle u-2v+4w, u-2v+4w\rangle$

$= \langle u,u\rangle - 2\langle u,v\rangle + 4\langle u,w\rangle$

$-2\langle v,u\rangle + (-2)^2\langle v,v\rangle - 2\cdot 4\langle v,w\rangle$

$+4\langle w,u\rangle + 4\cdot(-2)\langle w,v\rangle + 16\langle w,w\rangle$

$= 1 - 2\cdot 2 + 4\cdot 5 - 2\cdot 2 + 4\cdot 2^2 - 8\cdot(-3)$

$+4\cdot 5 - 8\cdot(-3) + 16\cdot 7^2 = 881$

Oppg 25

$\|u+v\|^2 + \|u-v\|^2 = \langle u+v, u+v\rangle$
$+\langle u-v, u-v\rangle$

$= 2\langle u,u\rangle + 2\langle v,v\rangle = 2\|u\|^2 + 2\|v\|^2$

Parallellogram-likningen!

Oppg 26

$$\|u+v\|^2 - \|u-v\|^2 =$$

$$\langle u+v, u+v \rangle - \langle u-v, u-v \rangle$$

$$= \langle u,u \rangle + 2\langle u,v \rangle + \langle v,v \rangle$$

$$- \langle u,u \rangle + 2\langle u,v \rangle - \langle v,v \rangle$$

$$= 4\langle u,v \rangle$$

QED

Oppg 30

a) $$\langle f,g \rangle = \int_0^1 \cos 2\pi x \, \sin 2\pi x \, dx$$

$$= \int_0^1 \frac{1}{2} \sin 4\pi x \, dx = \left[-\frac{1}{8\pi} \cos(4\pi x) \right]_0^1$$

$$= 0.$$

b) $$\langle f,g \rangle = \int_0^1 x e^x dx = \left[(x-1)e^x \right]_0^1 = 1$$

Oppg 34 | Oppg 34

a)

$\langle p, q \rangle = 1 \cdot 2 + 3 \cdot 0 + (-5)(-3) = 17$

$\langle p, p \rangle = 1^2 + 3^2 + (-5)^2 = 35$

$\langle q, q \rangle = 2^2 + 0^2 + (-3)^2 = 13$

so

$$\cos\theta = \frac{\langle p, q \rangle}{\|p\| \, \|q\|} = \frac{17}{\sqrt{35}\sqrt{13}}$$

Oppg 39 Vi får 3 likninger

$\vec{u} \cdot \vec{v} = 0 \quad \vec{u} \cdot \vec{w} = 0 \quad \vec{v} \cdot \vec{w} = 0$

$$\left.\begin{array}{l} k(-3) + 3 \cdot 1 + 2 \cdot l = 0 \\ k(-5) + 3 \cdot 5 + 2 \cdot 1 = 0 \\ (-3)(-5) + 1 \cdot 5 + l \cdot 1 = 0 \end{array}\right\} \Leftrightarrow \begin{array}{l} -3k + 2l = -3 \\ -5k = -17 \\ l = -20 \end{array}$$

dvs $l = -20 \quad k = 17/5 \quad (-3)\,17/5 + 2(-20) = -3$

Motsigelse.

Oppg 40 Vi skal løse likningen

$\|k\vec{u} + \vec{v}\|^2 = 13^2$ dvs

$\langle k\vec{u} + \vec{v}, k\vec{u} + \vec{v} \rangle = 169$

$k^2 \langle \vec{u}, \vec{u} \rangle + 2k \langle \vec{u}, \vec{v} \rangle + \langle \vec{v}, \vec{v} \rangle = 169$

$3k^2 + 56k + 310 = 169$

$3k^2 + 56k + 141 = 0$

$$k = \frac{-56 \pm \sqrt{56^2 - 4 \cdot 3 \cdot 141}}{6} = \frac{-56 \pm \sqrt{1444}}{6}$$

$$= \frac{-28 \pm 19}{3} = -47/3 \text{ eller } -3$$

Oppg 49 $\quad \|\vec{u}\| = \|\vec{v}\| = 1 \quad \vec{u} \perp \vec{v}$, så

$$\langle \vec{u}, \vec{u} \rangle = \langle \vec{v}, \vec{v} \rangle = 1 \quad \langle \vec{u}, \vec{v} \rangle = 0$$

$$\|\vec{u} + \vec{v}\|^2 = \langle \vec{u}, \vec{u} \rangle + 2\langle \vec{u}, \vec{v} \rangle + \langle \vec{v}, \vec{v} \rangle$$

$$= 1 + 0 + 1 = 2$$

så $\|\vec{u} + \vec{v}\| = \sqrt{2}$

Oppg 61

Følger av Cauchy-Schwarz.

Oppg 62

$$\langle f_k, f_l \rangle = \int_0^{\pi} \sin kx \, \sin lx \, dx$$

Formel: $\sin A \cdot \sin B = \frac{1}{2} \cos(A-B) - \frac{1}{2} \cos(A+B)$

$$\langle f_k, f_l \rangle = \frac{1}{2} \int_0^{\pi} \cos(k-l)x - \cos(k+l)x \, dx$$

$$= \frac{1}{2} \left[\frac{+\sin(k-l)x}{k-l} + \frac{\sin(k+l)x}{k+l} \right]_{x=0}^{x=\pi} = 0$$

for $k \neq l$.