Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
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Midterm in MA1301-Number theory

Friday October 7, 2005
Time: 08:15 – 9:45

No aids permitted.

The first part of the problem set has two problems; give detailed answers to these questions. Part two consists of multiple choice problems. Make a circle around the correct answer, and hand in the sheet. Be clear; if it is difficult to see what alternative you have chosen, you will not get any points for the problem.

Part 1:

Problem 1

a) What is the greatest common divisor of two positive numbers?

b) What is the condition for the diofantic equation $ax + by = c$ to have integer solutions?

You are to put stamps of a total value of 32 kroner on a letter, and have stamps of value 7.50 kroner and 9.50 kroner available.

c) Write down a diofantic equation which shows the problem. Solve the equation.

d) Find all possible combinations of stamps that you can put on the letter.

Problem 2 What is a prime? Show that there are infinitely many different primes.

Part 2: Candidate number:

Problem 3

a) Which of the alternatives shows the value of these binomial coefficients (make a circle):

$\binom{5}{3}$	-3	1	15	10	5
$\binom{10}{7}$	84	93	120	70	7

b) Decide whether these statements are correct (make a circle):

$\binom{n}{1} = n$	Right	Wrong
$\binom{n}{2} = n(n+1)$	Right	Wrong
$\binom{n}{k} = \binom{n}{n-k}$	Right	Wrong
$\binom{n}{k} = -\binom{n}{n-k}$	Right	Wrong
$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$	Right	Wrong
$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k-1}$	Right	Wrong

Problem 4

a) Compute the greatest common divisor of the following pairs of numbers (make a circle):

$gcd(17, 3)$	-1	3	1	17	51
$gcd(2883, 219)$	1	13	3	97	73
$gcd(55, 89)$	1	4	11	23	3

b) Decide whether the following diofantic equations have integer solutions (make a circle):

$18x + 42y = 1$	Has solutions	Does not have solutions
$18x + 42y = 2$	Has solutions	Does not have solutions
$18x + 42y = 3$	Has solutions	Does not have solutions
$18x + 42y = 6$	Has solutions	Does not have solutions
$18x + 42y = 30$	Has solutions	Does not have solutions
$18x + 42y = -78$	Has solutions	Does not have solutions

Problem 5 Decide whether the following statements are correct (make a circle):

There are infinitely many primes of the form $4n + 3$	Right	Wrong
There are infinitely many primes of the form $3n + 6$	Right	Wrong
There are integers a and b such that $\sqrt{3} = \frac{a}{b}$	Right	Wrong

Jon Eivind Vatne