Noen nyttige formler

De Moivres formel: $(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$.

Cauchy–Riemann-ligningene: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

Laplace-operatoren i to variable: $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

Noen komplekse funksjoner:

$$e^z = \exp(z) = e^x(\cos(y) + i\sin(y)),$$

$$\log(z) = \ln(|z|) + i\,\arg(z), \quad \mathrm{Ln}(z) = \mathrm{Log}(z) = \ln(|z|) + i\,\mathrm{Arg}(z),$$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}, \qquad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i}.$$

Cauchys generaliserte formel:

$$f^{(n)}(z) = \frac{n!}{2\pi i}\int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}}\,d\zeta.$$

Jordans lemma:

$$\int_0^\pi e^{-R\sin\theta}d\theta \le \frac{\pi}{R}.$$

Noen potensrekker:

$$\frac{1}{1-z} = \sum_{n=0}^\infty z^n = 1 + z + z^2 + z^3 + \cdots$$

$$e^z = \sum_{n=0}^\infty \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

$$\cos(z) = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!}z^{2k} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \cdots$$

$$\sin(z) = \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)!}z^{2k+1} = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots$$

Noen trigonometriske identiteter:

$$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v), \qquad \cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v),$$

$$\sin(2u) = 2\sin(u)\cos(u), \qquad \cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u),$$

$$2\sin(u)\cos(v) = \sin(u-v) + \sin(u+v), \qquad 2\cos(u)\cos(v) = \cos(u-v) + \cos(u+v),$$

$$2\sin(u)\sin(v) = \cos(u-v) - \cos(u+v).$$

Fourierrekker for en periodisk funksjon med periode $2L$:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L} = a_0 + \sum_{n=1}^\infty \left(a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right)$$

$$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L}\, dx$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx, \qquad a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx,$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx.$$

Cosinus- og sinusrekker for en funksjon definert på $[0, L]$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \qquad a_0 = \frac{1}{L} \int_{0}^{L} f(x)\, dx,$$

$$a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx.$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \qquad b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx.$$

Noen integraler:

$$\int x^n e^x\, dx = x^n e^x - n \int x^{n-1} e^x\, dx$$

$$\int x^n \ln(x)\, dx = \frac{x^{n+1}}{n+1} \ln(x) - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int x^n (\ln(x))^m\, dx = \frac{x^{n+1}}{n+1} (\ln(x))^m - \frac{m}{n+1} \int x^n (\ln(x))^{m-1}\, dx$$

$$\int e^{ax} \cos(bx)\, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos(bx) + b \sin(bx)) + C$$

$$\int e^{ax} \sin(bx)\, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin(bx) - b \cos(bx)) + C$$

$$\int x^m \cos(bx)\, dx = \frac{x^m \sin(bx)}{b} - \frac{m}{b} \int x^{m-1} \sin(bx)\, dx$$

$$\int x^m \sin(bx)\, dx = -\frac{x^m \cos(bx)}{b} + \frac{m}{b} \int x^{m-1} \cos(bx)\, dx$$