MA2501 Numeriske metoder

Øving 4

Guidance: 9/2, 08.15 - 10.00

Since the number of exercises is quite extensive, give priority to the exercises not marked with $\star$ in the first place.

Exercise 1

p. 161–162, **problem 1, 3, 4,** and p. 165, **problem 27**

Exercise 2

p. 178, **problem 13**.

Exercise 3

p. 179, **computer problem 10**.
Plot $\Pi_{i=0}^{n}(x - x_i)$, $-1 \leq x \leq 1$, for the three cases:

$$\begin{array}{ll} x_i = \cos[(2i+1)\pi/(2n+2)], & i = 0, \ldots, n \\ x_i = \cos(\pi i/n), & i = 0, \ldots, n \\ x_i = -1 + 2i/n, & i = 0, \ldots, n \end{array}$$

You can take advantage of the attached function `w.m`. In that case, you will (for example) obtain the plot by means of

```
>> n=3
>> i=[0:n]
>> x= cos((2*i+1)*pi/(2*n+2))
>> x(x,-1,1,'b')
```

Hint: If you want to make several plots in the same figure, you can make use of the `hold`-comand.

Exercise 4

p. 179, **computer problem 1,2**. Use the function `polyval` and `polyfit` in Matlab. Make a plot of $f(x)$ and $p(x)$. In the book, they use 21 nodes. In addition, make a try with 6 and 11 nodes.

Exercise 5*

Given $n+1$ distinct nodes, $x_0, x_1, \ldots, x_n$. Find a polynomial of least degree that satisfy

$$p(x_i) = y_i, \qquad p'(x_i) = v_i, \qquad i = 0, 1, \ldots, n. \tag{1}$$

This type of interpolation is called *Hermite-interpolation*.

- A reasonable assumption is that $p(x)$ would be of degree $2n+1$ or less. Why?

- Show that
$$p(x) = \sum_{i=0}^{n} y_i A_i(x) + \sum_{i=0}^{n} v_i B_i(x)$$
will satisfy (1) if $A_i(x)$ and $B_i(x)$ are given by
$$\begin{aligned} A_i(x_j) &= \delta_{ij}, \qquad & B_i(x_j) &= 0, \\ A_i'(x_j) &= 0, & B_i'(x_j) &= \delta_{ij}, \end{aligned}$$
where $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$.

- Let $\ell_i(x),\ i = 0, \ldots, n$ be the cardinal functions. Prove that the following polynomial satisfies the conditions above:
$$\begin{aligned} A_i(x) &= [1 - 2(x - x_i)\ell_i'(x_i)]\ell_i^2(x), \qquad & i = 0, \ldots, n, \\ B_i(x) &= (x - x_i)\ell_i^2(x). \end{aligned}$$

- Use this to find a polynomial of degree three that satisfies $p(1) = 1,\ p(2) = 14,\ p'(1) = 4,\ p'(2) = 24$.

Exercise 6*

a) Assume that you have found a polynomial $p_n(x)$ of degree $\leq n$, that interpolates the points $(x_i, y_i)_{i=0}^{n}$. Suppose that you wish to add an extra point (x_{n+1}, y_{n+1}). All nodes are distinct.

   Find the polynomial q, such that the new interpolation polynomial is written
$$p_{n+1}(x) = p_n(x) + q(x).$$

b) Use the previous result .... to find the polynomials p_0, p_1 and p_3 that interpolates ...

x	1	2	3/2
y	1	5	11/4