MA2501 Numerical Methods

Assignment 8

Tutorials: 16/3 og 23/3.

Exercise 1

Given the linear system of equations $A\mathbf{x} = \mathbf{b}$. For this system, let $\mathbf{x}$ be the exact solution, and $\tilde{\mathbf{x}}$ an approximate solution.

a) See p. 274 and p. 340 in C&K. Prove that

$$\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|} \leq \kappa(A)\frac{\|\mathbf{r}\|}{\|\mathbf{b}\|}$$

where $\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}}$ and $\mathbf{r} = A\tilde{\mathbf{x}} - \mathbf{b}$.

b) Chapter 7.1, problem 5. Find also the condition number of the matrix, and compare your results with the inequality from a).

c) (MATLAB) The purpose of this exercise is to demonstrate that truncation errors can be a serious problem even for the direct solution of some linear system, if the system is badly conditioned.

The Hilbert matrix is an $n \times n$ matrix, having the elements $a_{ij} = 1/(i+j-1)$. Let A be an $n \times n$ Hilbertmatrix, $\mathbf{x}$ some vector of your own choice, but of dimension n, and let $\mathbf{b} = A\mathbf{x}$. Thus you have an system of equations of which you know the exact solution. Solve the problem, using MATLABs solver. You then get some *approximate* solution $\tilde{\mathbf{x}}$. What is the size of the error vector and the residual vector, measured in max-norm? Find also the condition number of the matrix.

Try this for $n = 5$, $n = 10$ and $n = 15$.

Repeat the experiment using some arbitrary matrix. Do you get the same results?

Some useful MATLAB-commands:
`hilb(n)`: Makes $n \times n$ Hilbert matrise.
`norm(x,inf)`: The max-norm of a vector.
`cond(A,inf)`: The condition number of A, using the max-norm.

Exercise 2

Chapter. 7.2, problem 9, Chapter 8.1 problem 4.

Oppgave 3

Chapter 8.2, problems 3-9.
(NB! The answer of problem 9 is wrong.)

Oppgave 4

Given the system of equations on page 262 in C&K. If you try to solve this using Jacobi, Gauss-Seidel or SOR iterations, will the iterations converge?

Oppgave 5

Chapter 8.2, computer problems 3 and 4.