MA2501 Numerical methods

Problem set 1

Instruction: 19th and 26th of January, 2006

Problem 1

The following functions and domains will be used in this problem.

$$
\begin{aligned}
f_1(x) &= x^3 - 3x + 1, && \text{on } [0,1] \\
f_2(x) &= \cos(x) - \cos(3x), && \text{on } [1,2] \\
f_3(x) &= x^3 - 6x^2 + 12x - 8, && \text{on } [1.5,3]
\end{aligned}
$$

We will study their zeros and ways of constructing these zeros.

a) Prove that each of the functions $f_1(x)$, $f_2(x)$, and $f_3(x)$ have at least one zero within their respective interval. Are the zeros unique?

b) Find the zeros of $f_2(x)$ using

i) the bisection method (intervallhalvveringsmetoden)

ii) Newton's method. Use a starting value x_0 in the middle of the interval.

Perform 3–4 iterations (by hand).

c) Find the zeros of $f_1(x)$, $f_2(x)$, and $f_3(x)$ by means of the MATLAB scripts `bisection.m` and `newton.m` (available from the course homepage). The scripts implement the solution to $f_1(x) = 0$ and must be modified to solve the other equations.

Pay particular attention to how the iterations converge in each case. Record (on paper) how many iterations are needed to achieve 8 correct digits. Is this in agreement with general theory?

d) Improve `newton.m`. You may choose the level of sophistication of the improved code yourself. However, as a general guideline, the iteration should terminate when the computed solution is sufficiently accurate. See the pseudo code on pages 106–107 in Cheney and Kincaid for an example of how this may be defined.

e) Find the zeros using the MATLAB function `fzero`.

You may find the MATLAB function `fprintf` useful.

Problem 2

Solve problems **3.3.14** and **3.3.15** on page **134** of Cheney and Kincaid, *Numerical Mathematics and Computing*, 5th edition.

These problems provide a theoretical background on some of the aspects of the important *fixed point method* (also known as the *functional iteration method*) and will be covered in one of the lectures.