MA2501 Numerical methods

Problem set 2

Instructions: 26th of January and 2nd of February

This problem set is perhaps a bit too extensive. You should prioritise solving the problems ***not*** marked by an asterisk (*).

Problem 1

The equation

$$f(x) = x - 0.99(e^x - 1) = 0 \tag{1}$$

has two solutions, $x = 0$ and $x = \alpha > 0$.

a) Find α by means of Newton's method. Draw a graph of $f(x)$ and use this to find a suitable starting value x_0.

Hint: A suitable graph of (1) can be constructed in MATLAB using the statements

```
f = inline('x - 0.99*(exp(x) - 1)')
fplot(f, [-0.02, 0.04])
grid on
```

b) Write a fixed point iteration scheme (fikspunktiterasjonsskjema)—not Newton's method—that will converge to α. Calculate the maximum number of iterations needed to compute the fixed point with an absolute error less than 10^{-6}.

Hint: You may once more find drawing a graph of the functions helpful in determining the location of α and how to choose the starting value x_0. You may have to rewrite the fixed point scheme to achieve convergence.

Problem 2

a) Solve problem **3.2.20** on page **118**. Explain what happens in each of the three cases. Drawing a graph and performing a few iterations may be helpful.

b) Solve problem **3.2.33** on page **119**.

Problem 3

a) Assume that $f(r) = 0 = f'(r)$ but $f''(r) \neq 0$ for some $r \in \mathbb{R}$. Moreover, assume that f, f' and f'' are continuous in a neighbourhood of r. Show that it is possible to select $\omega \in \mathbb{R}$ such that a modified version of Newton's method, given by

$$x_{n+1} = x_n - \omega \frac{f(x_n)}{f'(x_n)},$$

converges quadratically to r. Find ω, implement the method in MATLAB, and test the resulting programme on an equation of your own choosing.

Hint: expand $f(x_n)$ and $f'(x_n)$ in Taylor series around r.

b)* Solve "Computer problem" **3.3.12** on page **135**. Use a test equation of your own choosing.

Problem 4

Solve "Computer problem" **3.2.23** on page **123**. Use starting value $(1, 1)$ in problems **a)** and **b)**. You do not have to solve all of the problems, but at least problems **a)** and **c)** are recommended.

Problem 5*

A spherical, undeformable ball floats in water. The ball is made of a homogenous material having density 60% of that of water. How deep will the ball sink into the water?

Suggested solution procedure

- Make an illustration of the situation.
- Determine which principles govern the physical process.

- Create a mathematical model of the system.
- Solve the mathematical model as well as possible. You may need to employ computer software such as MATLAB.

Remark: These kinds of open problems are the most common in applied mathematics. Solving them requires knowledge outside of the subject matter of a particular course. The problem's intent is partly to inspire independent work and partly to contextualize the themes presented in this course.