MA2501 Numerical methods

Mandatory problem set 1

Practical information

- **Handout**: Monday the 20th of February
- **Deadline**: Monday the 13th of March
- **Guidance**:
  - The lecturer will be in office 1354 of "Sentralbygg II" during regular lecture hours and scheduled exercise guidance hours. Additionally, from 2.15pm to 4pm every day during the week of 27th of February to 3rd of March (i.e. week 9), the lecturer will be in office 1354 as well. Questions may be sent by email to `<bardsk@math.ntnu.no>` and will be answered as far as the questions are reasonable.
  - The exercise assistant will be in computer lab 'Vegas' (room 212 of "Sentralbygg II") from 12.15pm to 2pm every day during the week of 6th of March to 10 of March (week 10). He will also answer reasonable questions sent by email to

    `<evensber@stud.math.ntnu.no>`

  All email sent to the course staff *must* be marked 'MA2501'.
- **Size of groups**: At most three students may constitute a group.

Introduction

This problem set deals with practical use of spline functions. Chapters **9.1** and **9.2** of Kincaid and Cheney contain necessary background material. The problem set entails MATLAB programming, so you should know how to write

and document MATLAB functions. The code you write should be documented using comment statements and be as structured and readable as possible.

You are encouraged to use MATLAB's own spline function, known as `spline`, in your programming work. In this case you should carefully read the function's documentation to fully comprehend its features and limitations.

We recommend that the minimum versions of the problems be resolved initially and, moreover, that the solution to this version be as simple as possible. The additional problems may be resolved subsequently. Be aware, though, that resolving *all* of the additional problems is a significant amount of work. You are hence not expected to solve them all. However, solving most of the additional problems satisfactorily is a definite advantage.

The report

Every group will produce an independent report from the work. The size of the report should be no more than $\mathbf{3 + 2n}$ pages with $\boldsymbol{n}$ being the number of students in the group. The limitation does *not* include print-out of MATLAB code. You may discuss different approaches to solving the problems with other groups, but the discussion should not infringe on the independence of your work. You are not to copy other people's MATLAB code and every member of the group should be able to defend the content of the report.

Please cite references if gleaning material from the internet or texts other than Kincaid and Cheney.

The report should be marked with a ***group number*** and the ***names*** of ***all*** members of the group. The group number will be assigned by the lecturer once the group has been formed. The report will at least treat the following points.

- **Problem** 1:
  - **1**) A description of the problem.
  - **2**) A description of how the problem has been resolved.
  - **3**) A set of data and a plot of the parametric spline which interpolates the set.
  - **4**) A print-out of the code used to construct the parametric spline as well as an example of actual use of the code.
  - **5**) If you have employed more than one data set, you have to explain why and your observations as well.
- **Problem** 2:

1) A drawing or photograph of the chosen object.

2) The chosen data sets, which types of splines have been employed where and reasons for these choices.

3) A plot of the figure as well as a print-out of the MATLAB code.

The report should contain descriptions of what you have observed and learnt in both problems.

Problem 1

Write a MATLAB function to construct and evaluate a parametric (two dimensional) kubic spline. The example on page **408** of Kincaid and Cheney contains a more detaild description of the procedure involved.

The minimum solution to this problem shall include

- Input arguments $(x_i, y_i)_{i=0}^{n}$. This is the data set for which you are to construct an interpolating spline.
- Output arguments $(S_{x,j}, S_{y,j})_{j=0}^{m}$. This is a set of data providing values of the spline also in *between* the knots.

  The output will be used, amongst other things, to produce a plot of the spline.
- Help text. The function will include sufficient help text to enable people unfamiliar with the code itself to use the function without having to read the detailed MATLAB statements.
- A demonstration of the function on a set of data chosen by you.

The additional problems accompanying the minimum requirements are

- A test of the MATLAB function on 'difficult' sets of data. What do you observe? When does the spline behave desirably and when does it not?
- When constructing parametric splines, the data x and y is known while the knots t_i of the independent variable t must be chosen in some way. Kincaid and Cheney suggest using $t_i = i$ for all $i = 0, 1, \ldots, n$. Can you think of other, possibly better, ways of selecting the knots? Implement and test your ideas.
- Any other things you may think of...

Problem 2

Draw an object of your own chosing (e.g. a teddy bear, a person's outline, a flower or other things) by means of cubic splines. You will normally need more than a single spline.

The minimum version of this problem is a simple object with few details.

You may additionally test different varieties of splines in addition to the natural cubic splines, for example splines with known end-point derivatives or parametric splines. Changing the rôles of the independent and dependent variable to plot $x = S(y)$ rather than $y = S(x)$ may be advantageous. What other alternatives can you think of?

Truly adventurous students may try to create a small animation based on the original object. I this case, the MATLAB functions `getframe` and `movie` may prove useful.