29. e). From 29. a)

$$\sum_{\rho} f(1+i t-\rho) = \frac{\log t}{8} + O(1) \quad ; \ \rho = \beta + i\gamma$$

$$\underbrace{\sum_{\rho} f(1-\beta+i(t-\gamma))}_{} = \frac{\log t}{8} + O(1)$$

$$(\star) \ldots \sum_{\substack{\rho \\ \gamma-t\leq 6}} f(1-\beta+i(t-\gamma)) + \sum_{\substack{\rho \\ -6<\gamma-t\leq 6}} f(1-\beta+i(t-\gamma)) + \sum_{\substack{\rho \\ \gamma-t>6}} f(1-\beta+i(t-\gamma)) = \frac{\log t}{8} + O(1)$$

In the sum: $\sum_{\substack{\rho \\ \gamma-t\leq -6}} f(1-\beta+i(t-\gamma))$, we have $0<1-\beta<1$, and $|t-\gamma|\geq 6$

Using 29. d), we have $\sum_{\substack{\rho \\ \gamma-t\leq 6}} f(1-\beta+i(t-\gamma)) \leq 0$

Similarly $\sum_{\substack{\rho \\ \gamma-t>6}} f(1-\beta+i(t-\gamma)) \leq 0$

Then, in $(\star)$ we have: $\sum_{\substack{\rho \\ -6<\gamma-t\leq 6}} f(1-\beta+i(t-\gamma)) \geq \frac{\log t}{8} + O(1) \ldots (\star\star)$

Since $1-\beta>0 \Rightarrow$ using 29. b) $f(1-\beta+i(t-\gamma)) \leq 1$

In $(\star\star)$ we have

$$\frac{\log t}{8} + O(1) \leq \sum_{\substack{\rho \\ -6<\gamma-t\leq 6}} f(1-\beta+i(t-\gamma)) \leq \sum_{\substack{\rho \\ -6<\gamma-t\leq 6 \ (t-6<\gamma\leq t+6)}} 1 = N(t+6) - N(t-6)$$

$\Rightarrow$ For t large: $N(t+6) - N(t-6) \geq \frac{\log t}{8} + O(1) \geq \frac{\log t}{16}$

$$\Rightarrow N(t+6) - N(t-6) \gg \log t$$

∎