31) Using the Riemann–Von Mangoldt formula:

$$N(T) = \frac{T}{2\pi}\log T + O(T) \quad , \text{ as } T \to \infty$$

$$\Rightarrow \underbrace{N(\gamma_n)}_{} = \frac{\gamma_n}{2\pi}\log \gamma_n + O(\gamma_n) \quad , \text{ as } n \to \infty$$

$$(\bigstar) \ldots \; n = \frac{\gamma_n}{2\pi}\log\gamma_n\left(1 + O\left(\frac{1}{\log\gamma_n}\right)\right).$$

Recall that $\gamma_n \xrightarrow[n\to\infty]{} \infty$

$$\Rightarrow \frac{2\pi n}{\gamma_n \log \gamma_n} = 1 + O\left(\frac{1}{\log\gamma_n}\right) \Rightarrow \frac{2\pi n}{\gamma_n \log\gamma_n} \longrightarrow 1$$

In $(\bigstar)$:

$$\log n = \log\gamma_n + \log\log\gamma_n - \log 2\pi + \log\left(1 + O\left(\frac{1}{\log\gamma_n}\right)\right)$$

$$\log n = \log\gamma_n + \log\log\gamma_n - \log 2\pi + O\left(\frac{1}{\log\gamma_n}\right)$$

$$\frac{\log n}{\log\gamma_n} = 1 + \frac{\log\log\gamma_n}{\log\gamma_n} - \frac{\log 2\pi}{\log\gamma_n} + O\left(\frac{1}{\log^2\gamma_n}\right)$$

$$\Rightarrow \lim_{n\to\infty}\frac{\log n}{\log\gamma_n} = 1 \Rightarrow \lim_{n\to\infty}\frac{\log\gamma_n}{\log n} = 1$$

also we have proved $\lim_{n\to\infty}\frac{2\pi n}{\gamma_n\log\gamma_n} = 1$

$$\Rightarrow \lim_{n\to\infty}\frac{2\pi n}{\gamma_n \log n} = 1 \Rightarrow \gamma_n \sim \frac{2\pi n}{\log n} \quad n\to\infty$$