33) From the Proposition in class 7

$$\frac{\zeta'}{\zeta}(\sigma+it) = \sum_{|t-\gamma|\le 1} \frac{1}{s-\rho} + O(\log t)$$

$$\left|\frac{\zeta'}{\zeta}(\sigma+it)\right| \le \sum_{|t-\gamma|\le 1} \frac{1}{|\sigma+it-(\frac{1}{2}+i\gamma)|} + C\log t$$

We are assuming RH, $\rho = \frac{1}{2}+i\gamma$

$$\left|\frac{\zeta'}{\zeta}(\sigma+it)\right| \le \sum_{|t-\gamma|\le 1} \frac{1}{|\sigma-\frac{1}{2}+i(t-\gamma)|} + C\log t$$

$$\left|\frac{\zeta'}{\zeta}(\sigma+it)\right| \le \sum_{|t-\gamma|\le 1} \frac{1}{\sigma-\frac{1}{2}} + C\log t$$

$$\le \frac{1}{\sigma-1/2} \sum_{|t-\gamma|\le 1} 1 + C\log t$$

↳ corollary in class

$$\le \frac{1}{\sigma-1/2}\,(C\log t) + C\log t$$

↳ $1 \le \frac{2}{\sigma-1/2}$; because $\frac{1}{2} < \sigma \le 2$

$$\le \frac{C\log t}{\sigma-1/2} + \frac{2C\log t}{\sigma-1/2}$$

$$= 3C\frac{\log t}{\sigma-1/2}$$

$$\Rightarrow \frac{\zeta'}{\zeta}(\sigma+it) = O\left(\frac{\log t}{\sigma-1/2}\right)$$