$6 < 0 < 2\pi$

•) $\xi(s) = \frac{1}{2} s(s-1) \pi^{-s/2} \Gamma\left(\frac{s}{2}\right) \zeta(s)$

•) $\xi(s) = e^{A+Bs} \prod_\rho \left(1 + \frac{s}{\rho}\right) e^{s/\rho}$

$$\left| \frac{\xi\left(\frac{1}{2}+it\right)}{\xi\left(-\frac{3}{2}+it\right)} \right| = \left| \frac{e^{A+B\left(\frac{1}{2}+it\right)} \cdot \prod_\rho \left(1 - \frac{\frac{1}{2}+it}{\rho}\right) e^{\frac{\frac{1}{2}+it}{\rho}}}{e^{A+B\left(-\frac{3}{2}+it\right)} \prod_\rho \left(1 - \frac{-\frac{3}{2}+it}{\rho}\right) e^{\frac{-\frac{3}{2}+it}{\rho}}} \right|$$

$$= \left| e^{2B} \prod_\rho \frac{\rho - \left(\frac{1}{2}+it\right)}{\rho - \left(-\frac{3}{2}+it\right)} \cdot e^{2/\rho} \right|$$

$$= e^{2B} \cdot \prod_\rho \left| \frac{\rho - \left(\frac{1}{2}+it\right)}{\rho - \left(-\frac{3}{2}+it\right)} \right| \cdot e^{\sum_\rho \left[2 \operatorname{Re}\left\{\frac{1}{\rho}\right\}\right]}$$

$$\sum_\rho \operatorname{Re}\left\{\frac{1}{\rho}\right\} = -B$$

$$= e^{2B} \prod_\rho \left| \frac{\rho - \left(\frac{1}{2}+it\right)}{\rho - \left(-\frac{3}{2}+it\right)} \right| \cdot e^{2 \sum_\rho \operatorname{Re}\left\{\frac{1}{\rho}\right\}}, \quad e^{-B}$$

$$= \prod_\rho \left| \frac{\frac{1}{2}+i\gamma - \left(\frac{1}{2}+it\right)}{\frac{1}{2}+i\gamma - \left(-\frac{3}{2}+it\right)} \right| = \prod_\rho \left| \frac{i(\gamma - t)}{2 + i(\gamma - t)} \right|$$

$$\log \left| \xi\left(\frac{1}{2}+it\right) \right| - \log \left| \xi\left(-\frac{3}{2}+it\right) \right| = \sum_\rho \log \left| \frac{\gamma - t}{2 + i(\gamma - t)} \right|$$

$$= \frac{1}{2} \sum_\rho \log \left(\frac{(\gamma - t)^2}{4 + (\gamma - t)^2} \right)$$

$$= -\frac{1}{2} \sum_\gamma f(\gamma - t)$$

$\log|\xi(\frac{1}{2}+it)| - \log|\xi(-\frac{3}{2}+it)|$

$-\log|\Gamma(\frac{5}{2}-it)|$

$\xi(s) = \xi(1-s)$ (2)

$= \log|\xi(\frac{1}{2}+it)| - \log|\xi(\frac{5}{2}-it)|$

$\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s)$

$\log|\xi(\sigma+it)| = \log(\frac{1}{2}) + \ln|s| + \ln|s-1| + (-\frac{\mathrm{Re}\,s}{2})\log\pi + \log|\Gamma(\frac{s}{2})| + \log|\zeta(s)|$

$\Rightarrow \log|\xi(\sigma+it)| = \log(\frac{1}{2}) + \log|\sigma+it| + \log|\sigma-1+it| - \frac{\sigma}{2}\log\pi + \log|\Gamma(\frac{\sigma+it}{2})| + \log|\zeta(\sigma+it)|$

$t \to \infty$: $\log|\sigma+it| = \frac{1}{2}\log(\sigma^2+t^2) = \frac{1}{2}\log\left|t^2(1+\frac{\sigma^2}{t^2})\right| = \log t + \frac{1}{2}\log(1+\frac{\sigma^2}{t^2})$

$= \log t + O(\frac{1}{t^2})$

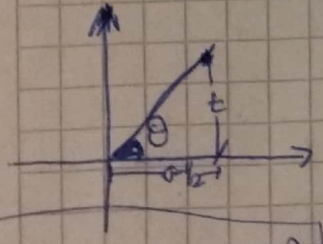
$\log|\Gamma(\frac{\sigma+it}{2})| =$

$\log\Gamma(s) = (s-\frac{1}{2})\log s - s + c + O(\frac{1}{|s|})$

$\log|\Gamma(s)| = \mathrm{Re}\{(\sigma-\frac{1}{2})\log s\} - \mathrm{Re}(s) + c + O(\frac{1}{|s|})$

$\log|\Gamma(\sigma+it)| = \mathrm{Re}\{(\sigma-\frac{1}{2}+it)\log(\sigma-\frac{1}{2}+it)\} - \sigma + c + O(\frac{1}{|\sigma+it|})$

$= (\sigma-\frac{1}{2})\cdot\log|\sigma-\frac{1}{2}+it| - t\cdot\arg(\sigma-\frac{1}{2}+it) - \sigma + c + O(\frac{1}{t})$



$= (\sigma-\frac{1}{2})\log|\sigma-\frac{1}{2}+it| - t\cdot\arg(\sigma-\frac{1}{2}+it) - \sigma + c + O(\frac{1}{t})$

$= (\sigma-\frac{1}{2})\cdot\{\log t + O(\frac{1}{t^2})\} - t\cdot\arctan(\frac{t}{\sigma-\frac{1}{2}}) - \sigma + c + O(\frac{1}{t})$

$\arctan x = \frac{\pi}{2} + O(\frac{1}{x})$

$= (\sigma-\frac{1}{2})\log t - t\{\frac{\pi}{2} + O(\frac{1}{t})\} - \sigma + c + O(\frac{1}{t})$

$\log|\Gamma(\sigma+it)| = (\sigma-\frac{1}{2})\log t - t\cdot\frac{\pi}{2} + O(1)$

$$\log|\xi(\sigma+it)| = \log\left(\tfrac{1}{2}\right) + 2\log t + O\left(\frac{1}{t^2}\right) - \frac{\sigma}{2}\log\pi + \ln|\zeta(\sigma+it)| + \left(\frac{\sigma}{2}-\frac{1}{2}\right)\log\left(\frac{t}{2}\right) + \frac{t}{2}\cdot\frac{\pi}{2} + O(1)$$

$$\boxed{\log|\xi(\sigma+it)| = \left(\frac{\sigma}{2}-\frac{1}{2}\right)\log t - \frac{t}{2}\cdot\frac{\pi}{2} + 2\log t + \log|\zeta(\sigma+it)| + O(1)}$$

$$\log\left|\xi\left(\tfrac{1}{2}+it\right)\right| - \log\left|\xi\left(\tfrac{5}{2}-it\right)\right|$$

$$= \log\left|\xi\left(\tfrac{1}{2}+it\right)\right| - \log\left|\xi\left(\tfrac{5}{2}+it\right)\right|$$

$$= \left(\frac{1/2}{2}-\frac{1}{2}\right)\log t - \frac{t}{2}\cdot\frac{\pi}{2} + 2\log t + \log\left|\zeta\left(\tfrac{1}{2}+it\right)\right| + O(1)$$

$$-\left(\frac{5/2}{2}-\frac{1}{2}\right)\log t + \frac{t}{2}\cdot\frac{\pi}{2} - 2\log t - \log\left|\zeta\left(\tfrac{5}{2}+it\right)\right| + O(1)$$

$$= -\log t + \log\left|\zeta\left(\tfrac{1}{2}+it\right)\right| + O(1)$$

$$\log\left|\zeta\left(\tfrac{5}{2}+it\right)\right|$$

$$\log\zeta(\sigma+it) = \sum_{n\geq 2}\frac{\Lambda(n)}{n^{\sigma}\log n}, \quad \sigma > 1$$

$$\log\left|\zeta(\sigma+it)\right| = \left|\operatorname{Re}\left\{\sum_{n\geq 2}\frac{\Lambda(n)}{n^{\sigma+it}\log n}\right\}\right|$$

$$\log\left|\zeta\left(\tfrac{5}{2}+it\right)\right| \leq \left|\sum_{n\geq 2}\frac{\Lambda(n)}{n^{5/2+it}\log n}\right| \leq \sum_{n\geq 2}\frac{\Lambda(n)}{n^{5/2}\log n} \leq \sum_{n\geq 2}\frac{1}{n^{5/2}} \leq \zeta(5/2)$$

$$\boxed{\Lambda(n) \leq \log n}$$

$$\Rightarrow \log\left|\zeta\left(\tfrac{5}{2}+it\right)\right| = O(1)$$

$$\log\left|\xi\left(\tfrac{1}{2}+it\right)\right| - \log\left|\xi\left(\tfrac{5}{2}-it\right)\right| = -\log t + \log\left|\zeta\left(\tfrac{1}{2}+it\right)\right| + O(1)$$

$$-\frac{1}{2}\sum_{\gamma} f(\gamma-t) = -\log t + \log\left|\zeta\left(\tfrac{1}{2}+it\right)\right| + O(1)$$

$$\log t - \frac{1}{2}\sum_{\gamma} f(t-\gamma) + O(1) = \log\left|\zeta\left(\tfrac{1}{2}+it\right)\right|$$