Prove that the Diophantine equation $y^2+2 = x^3$ has only the integer solutions $y = \pm 5, \quad x = 3$. [Hint: Use that $\mathbb{Z}[\sqrt{-2}]$ is a UFD.]