

Show that  $f(x) = x^3 - x + 1$  is irreducible over  $\mathbb{Q}$ . Let  $\alpha$  denote a root of  $f(x)$ . Write  $\frac{1}{\alpha^2 + 1}$  as  $g(\alpha)$ , where  $g(x) \in \mathbb{Q}[x]$ ,  $\text{degree}(g(x)) \leq 2$ .