

Exercise 1. In the category **Set**, show that epimorphisms and split epimorphisms are precisely the surjective functions.

Exercise 2. Let \mathcal{C} be a category.

1. Assume f is both a monomorphism and a split epimorphism. Show that f is an isomorphism.
2. Assume f is both a split monomorphism and an epimorphism. Show that f is an isomorphism.

Exercise 3. In the category **Top** of topological spaces, investigate which maps are monomorphisms, epimorphisms, and isomorphisms.

How does this change if we consider instead the category **Haus** of Hausdorff spaces?

(During the lecture I asked about topological spaces but was thinking about Hausdorff spaces — in any case both questions make sense.)

Exercise 4. Consider the construction sending each set to its power set. Does this define a functor from **Set** to **Set**? If so, is this functor full? Faithful? Dense?