Exercise 34. Calculate the following derived functors:

- $\mathrm{Tor}_n^{\mathbb{Z}}(\mathbb{Z}/(a), -)(\mathbb{Z}/(b))$ and $\mathrm{Tor}_n^{\mathbb{Z}}(-, \mathbb{Z}/(b))(\mathbb{Z}/(a))$.
- $\mathrm{Tor}_n^{\mathbb{Z}[x]/(x^2)}(\mathbb{Z}, -)(\mathbb{Z}/(2))$. (Here both $\mathbb{Z}$ and $\mathbb{Z}/(2)$ are modules over the ring $\mathbb{Z}[x]/(x^2)$ by letting x act as 0.)
- Consider the poset

$$(X, \leq) = \left\{ \begin{matrix} & \omega & \\ / & & \backslash \\ a & & b \\ \backslash & & / \\ & 0 & \end{matrix} \right\}.$$

For any $i \in X$ we denote by S_i the Ab-valued presheaf with

$$S_i(j) = \begin{cases} \mathbb{Z} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

For any pair i and j, calculate all $\mathrm{Ext}^n_{\mathrm{presh}_{\mathbf{Ab}} X}(S_i, S_j)$.

Exercise 35. Let $\mathcal{A}$ be an abelian category with enough injectives. Consider the poset $X = \{1 \leq 2\}$. Then presheaves on X are essentially morphisms, and we can consider Ker as a functor from $\mathrm{presh}_{\mathcal{A}} X$ to $\mathcal{A}$. Note that this functor is left exact.

Describe the right derived functors $\mathbb{R}^n\mathrm{Ker}\colon \mathrm{presh}_{\mathcal{A}} X \to \mathcal{A}$.

Exercise 36. Let $\mathtt{F}\colon \mathcal{A} \to \mathcal{B}$ be left exact, and $\mathcal{A}$ have enough injectives.

Assume there is a full subcategory $\mathcal{C}$ of $\mathcal{A}$ with the following properties:

- for any morphism f in $\mathcal{C}$, which is a monomorphism in $\mathcal{A}$, also the cokernel (calculated in the category $\mathcal{A}$) lies in $\mathcal{C}$;
- for any object A of $\mathcal{A}$, there is a monomorphism $A \to C$ to some object of $\mathcal{C}$;
- for any short exact sequence in $\mathcal{C}$ (that is a short exact sequence in $\mathcal{A}$ which lies entirely in the subcategory $\mathcal{C}$), the image of this sequence under $\mathtt{F}$ is also short exact.

Show that one can calculate $\mathbb{R}^n\mathtt{F}$ by using coresolutions with terms in $\mathcal{C}$ (rather than with injective terms, as in the definition of $\mathbb{R}^n\mathtt{F}$).