Exercise 37. Show that taking pushouts and pullbacks in Yoneda-Ext commutes:

If $0 \to B \to E \to A \to 0$ is an extension in $\mathrm{YExt}^1(A, B)$, and $f\colon A' \to A$ and $g\colon B \to B'$ are morphisms, then pullback along f of the pushout along g of the extension is the same as the pushout along g of the pullback along f.

Exercise 38. In an abelian category, assume we are given two short exact sequences $0 \to A \to B \to C \to 0$ and $0 \to C \to D \to E \to 0$ as in the first row and last column of the following diagram. Assume that $\mathrm{Ext}^2(E, A) = 0$.

Show that we can complete the diagram as indicated by the dashed arrows, such that also the middle row and column are short exact sequences.

$$
\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
 & & \| & & \vdots & & \downarrow & & \\
0 & \longrightarrow & A & \dashrightarrow & X & \dashrightarrow & D & \longrightarrow & 0 \\
 & & & & \vdots & & \downarrow & & \\
 & & & & E & = & E & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}
$$

Exercise 39. For a complex $X^\bullet$, we can make a new complex out of its homologies

$$\mathrm{H}^\bullet(X^\bullet) = \cdots \xrightarrow{0} \mathrm{H}^{-1}(X^\bullet) \xrightarrow{0} \mathrm{H}^0(X^\bullet) \xrightarrow{0} \mathrm{H}^1(X^\bullet) \xrightarrow{0} \cdots,$$

that is by setting all the maps to zero.

Let $\mathcal{A}$ be an abelian category. Show that the following two statements are equivalent.

- $\mathcal{A}$ is hereditary;
- for any complex $P^\bullet$ of projectives, there is a quasi-isomorphism $P^\bullet \to \mathrm{H}^\bullet(P^\bullet)$.