Exercise 40. Let $\mathcal{A}$ be an abelian category with enough projectives. Let $X = \{1 \leq 2\}$.

Show that

$$\operatorname{gldim} \operatorname{presh}_{\mathcal{A}} X = \operatorname{gldim} \mathcal{A} + 1.$$

Exercise 41. Let $R = \mathbb{Z}/(n)$ or $R = \mathbb{F}[X]/(f(X))$ for $\mathbb{F}$ a field.

Determine the global dimension of R, depending on n / $f(X)$.

(Hint: Remember the Chinese Remainder Theorem.)

Exercise 42. Let R be a ring. Show that

$$\operatorname{gldim} \operatorname{Mod} R = \sup\{\operatorname{pdim} R/I \mid I \text{ a right ideal}\}.$$

(Hint: Use Exercise 24.)