Exercise 9. Show that both the product and the coproduct of two abelian groups A and B given by the product $A \times B$ of the groups.

Exercise 10. In the category of groups, describe the product and coproduct (maybe harder) of two groups G and H.

Exercise 11. In the lecture I argued for a second definition of limit, but mainly argued that our original definition implies the new one. The aim of this exercise is to check that the second definition of limit also implies the first one:

Let F be a $\mathcal{C}$-valued presheaf on a poset $(X, \leq)$.

Assume L is an object of $\mathcal{C}$, together with morphisms $p_a \colon L \to F(a)$ for all $a \in X$, such that $F(b \leq a) \circ p_a = p_b$ whenever $b \leq a$.

Assume that for any object T together with morphisms $t_a \colon T \to F(a)$ such that $F(b \leq a) \circ t_a = t_b$ whenever $b \leq a$ there is a unique morphism $f \colon T \to L$ such that $t_a = p_a \circ f$.

Show that the functors

$$\mathrm{Hom}_{\mathcal{C}}(-, L) \text{ and } \mathrm{Hom}_{\mathrm{presh}_{\mathcal{C}}(X,\leq)}(\Delta -, F)$$

are naturally isomorphic.

Exercise 12. Let $\mathcal{C}$ be a category with precisely one object C. Assume the cardinality of $\mathrm{Hom}_{\mathcal{C}}(C, C)$ is finite.

Show that the product of C with itself exists if and only if $\mathrm{Hom}_{\mathcal{C}}(C, C) = \{\mathrm{id}_C\}$.