Exercise 13. Let $f\colon G \to H$ be a group homomorphism.

1. Let S be a subgroup of H. Find the pullback of

$$\begin{tikzcd} & S \arrow[d, hook, "\text{inclusion}"] \\ G \arrow[r, "f"] & H \end{tikzcd}$$

2. Let N be a normal subgroup of G. Find the pushout of

$$\begin{tikzcd} G \arrow[r, "f"] \arrow[d, two heads, "projection"'] & H \\ G/N & \end{tikzcd}$$

Exercise 14. Let $(X, \leq)$ be any poset, and F a Set-valued presheaf on X. Construct explicitly the limit and colimit of F. (In particular both limit and colimit exist.)

Exercise 15. In any category $\mathcal{C}$ we are given the solid part of the following diagram.

$$\begin{tikzcd} & B \prod_C X \arrow[r, dashed] \arrow[d, dashed] & X \arrow[d] \\ A \arrow[r] & B \arrow[r] & C \end{tikzcd}$$

Assume

- the pullback $B \prod_C X$ exists in $\mathcal{C}$, as indicated by the dashed arrows in the diagram above, and
- the pullback $A \prod_B (B \prod_C X)$ of the resulting left hand side of the diagram exists.

Show that the iterated pullback $A \prod_B (B \prod_C X)$ is the pullback of the large square

$$\begin{tikzcd} & X \arrow[d] \\ A \arrow[r] & C \end{tikzcd}$$

given by composing the two lower arrows.

Exercise 16. In a preadditive category $\mathcal{C}$, assume two objects C and D have a biproduct $C \oplus D$ as in the definition of additive categories.

Show that $C \oplus D$ is both the product and the coproduct of C and D.