Exercise 17. Let $f\colon A \to B$ be a morphism in an additive category, which has a kernel $k\colon K \to A$. Show that k is a monomorphism.

Exercise 18. Let $f\colon A \to B$ be a morphism in some abelian category. Assume that f can be written as the composition of a monomorphism $m\colon X \to B$ and an epimorphism $e\colon A \to X$. Show that X is the image of f.

Exercise 19. Consider a pullback square in any category (not assumed to be abelian or additive).

$$\begin{tikzcd} A \arrow[r, "a"] \arrow[d, "b"] & B \arrow[d, "c"] \\ C \arrow[r, "d"] & D \end{tikzcd}$$

Show: If d is mono then so is a.

Exercise 20 (3 by 3 Lemma, or 9 Lemma)**.** In an abelian category, consider the following commutative diagram with exact rows and columns.

$$\begin{tikzcd} & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\ & A \arrow[d] & B \arrow[d] & C \arrow[d] & \\ 0 \arrow[r] & D \arrow[r] \arrow[d] & E \arrow[r] \arrow[d] & F \arrow[r] \arrow[d] & 0 \\ 0 \arrow[r] & G \arrow[r] \arrow[d] & H \arrow[r] \arrow[d] & I \arrow[r] \arrow[d] & 0 \\ & 0 & 0 & 0 & \end{tikzcd}$$

Show that there is a short exact sequence $0 \to A \to B \to C \to 0$ fitting into the above diagram.

(You can choose if you want to consider the case of modules, so that you can use elements, or want to try the arguably more challenging approach of using the definition of abelian categories.)