Exercise 25.

1. Let P be a projective object in some abelian category $\mathcal{A}$. Let X be some poset, and $i \in X$. We construct an $\mathcal{A}$-valued presheaf F on X by

$$F(x) = \begin{cases} P & \text{if } x \leq i \\ 0 & \text{otherwise.} \end{cases}$$

Show that F is projective in $\operatorname{presh}_{\mathcal{A}} X$.

2. Let $\mathcal{A}$ be an abelian category with enough projectives, and let X be a finite poset. Show that also $\operatorname{presh}_{\mathcal{A}} X$ has enough projectives.

3. For each of the two **Ab**-valued presheaves of Exercise 5,

$$F = \begin{bmatrix} \mathbb{Z}/(2) & & \mathbb{Z}/(2) \\ & {\searrow}^{0} \quad {}^{1}{\swarrow} & \\ & \mathbb{Z}/(2) & \end{bmatrix} \text{ and } G = \begin{bmatrix} \mathbb{Z}/(2) & & 0 \\ & {\searrow}^{1} \quad {\swarrow} & \\ & \mathbb{Z}/(2) & \end{bmatrix},$$

find explicitly an epimorphism from a projective to the given presheaf.

Exercise 26. Show that

$$\operatorname{Hom}_{\mathbf{Ab}}(\mathbb{Z}/(m), \mathbb{Z}/(n)) = \mathbb{Z}/(\gcd(m,n)).$$

Exercise 27. Show directly (without using adjunction) that tensor products are right exact.