

Exercise 28. Calculate $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Z}/(n)$ and $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q}$.

Exercise 29. Show that tensor products are associative: Given a right R -module L , an R - S -bimodule M , and a left S -module N , show that

$$(L \otimes_R M) \otimes_S N \cong L \otimes_R (M \otimes_S N).$$

Exercise 30. Consider the poset

$$(X, \leq) = \left\{ \begin{array}{c} a \quad b \\ \diagdown \quad \diagup \\ 0 \end{array} \right\}.$$

Calculate all homologies of the complex

$$0 \rightarrow \begin{bmatrix} \mathbb{Z} & 0 & 0 \\ \swarrow & & \swarrow \\ & \mathbb{Z} & \end{bmatrix} \xrightarrow{\begin{smallmatrix} \cdot 2 & 0 \\ \cdot 3 \end{smallmatrix}} \begin{bmatrix} \mathbb{Z} & 0 & \mathbb{Z} \\ \swarrow & 1 & \swarrow \\ & \mathbb{Z} & \end{bmatrix} \xrightarrow{\begin{smallmatrix} 0 & \cdot 5 \\ 0 \end{smallmatrix}} \begin{bmatrix} \mathbb{Z} & & \mathbb{Z} \\ \swarrow & & \swarrow \\ & 0 & \end{bmatrix} \rightarrow 0$$

of \mathbf{Ab} -valued presheaves on (X, \leq) .