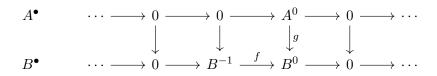
Exercise 31. Assume the following map of complexes is a quasi-isomorphism.



Show that $B^0 = B^{-1} \oplus A^0$, and that f and g correspond to the inclusions of the two summands into the sum.

Exercise 32. Let R be a ring. In the category $\mathbf{C}(\text{Mod } R)$ of complexes of right R-modules, we consider the complex given by putting R itself in degree 0, and 0 in all other degrees, and denote this complex by $R_{\text{in deg } 0}$. That is

$$R_{\text{in deg }0} = [\dots \to 0 \to R \to 0 \to \dots].$$

Show that

$$\operatorname{Hom}_{\mathbf{K}(\operatorname{ModR})}(R_{\operatorname{in} \operatorname{deg} 0}, -) \cong \mathrm{H}^{0}.$$

Exercise 33. Let $f^{\bullet}: A^{\bullet} \to B^{\bullet}$ be a morphism of complexes. Show that the morphism $\begin{bmatrix} id \\ 0 \end{bmatrix}: B^{\bullet} \to \operatorname{Cone}(f)$ is a *weak cokernel* of f^{\bullet} in the homotopy category.

(That is, the composition $\begin{bmatrix} \mathrm{id} \\ 0 \end{bmatrix} \circ f^{\bullet}$ is null-homotopic, and for any map of complexes $g^{\bullet} \colon B^{\bullet} \to C^{\bullet}$ such that $g^{\bullet} \circ f^{\bullet}$ is null-homotopic, g^{\bullet} factors through $\begin{bmatrix} \mathrm{id} \\ 0 \end{bmatrix}$ up to homotopy — the term "weak" refers to the fact that we do not require this factorization to be unique.)