Show that

1. $$\Lambda = k\left[\begin{tikzcd} 1 \arrow[r] \arrow[loop below] & 2 \arrow[r] \arrow[loop below] & \cdots \arrow[loop below] \arrow[r] & n \arrow[loop below] \end{tikzcd}\right] \Big/ (\text{arrows})^2$$

   is representation finite

2. $$\Lambda = k\left[\begin{tikzcd} 1 \arrow[r] \arrow[loop below] & 2 \arrow[loop below] & 3 \arrow[l] \arrow[r] \arrow[loop below] & 4 \arrow[loop below] \end{tikzcd}\right] \Big/ (\text{arrows})^2$$

   is not representation finite.

3. Find all (basic, up to iso) tilting modules over $\Lambda = k[1 \to 2 \to 3]$.

4. Let $$\Lambda = k\left[\begin{tikzcd} 1 \arrow[r, "a"] \arrow[d, "b"'] & 2 \arrow[d, "c"] \\ 3 \arrow[r, "d"'] & 4 \end{tikzcd}\right] \Big/ (ca - db),$$ and

   $$T = P_1 \oplus P_2 \oplus P_3 \oplus \tau^- P_4$$

   Show that T is tilting.

   Calculate:

   - $\Gamma = \operatorname{End}_\Lambda(T)^{op}$
   - $DT = \operatorname{Hom}(T, D\Lambda)$ as Γ-module
   - $\operatorname{Fac} T \subseteq \operatorname{mod} \Lambda$
   - $\operatorname{Sub} DT \subseteq \operatorname{mod} \Gamma$.

Find the AR-quiver (and in particular all quasi-simple modules) for

$\boxed{1}$ $$\Lambda = k\left[\begin{tikzcd} & 2 \arrow[dr] & \\ 1 \arrow[ur] \arrow[rr] & & 3 \end{tikzcd}\right]$$

$\boxed{2}$ $$\Lambda = k\left[\begin{tikzcd} 1 \arrow[ddr] & \\ 2 \arrow[dr] & \\ 3 \arrow[r] & 5 \\ 4 \arrow[ur] & \end{tikzcd}\right]$$

$\boxed{1}$ Let Λ be hereditary with indecomposable projectives $P_1, \dots, P_n$.

Assume $\text{length}_k\, \tau^{-i} P_j$ is a polynomial in i for all j.

a) Prove that the quiver of projectives is an extended Dynkin diagram.

b) Prove that the above polynomials are all linear.

$\boxed{2}$ Let $\Lambda = k[1 \rightrightarrows 2]$.

a) Let M be indecomposable. Give a condition on the dimension vector for M being preproj / preinj / regular.

b) Show that regular modules are <u>not</u> an abelian category.

$\boxed{1}$ Let Λ be a hereditary algebra.
Show that τ is left exact and τ^- is right exact.

$\boxed{2}$ Let Λ hereditary, M, N indecomp.
M preproj and N not preproj.
Show:

a) $\mathrm{Hom}(N, M) = 0$

b) $\mathrm{Ext}^1(M, N) = 0$

$\boxed{3}$ Let $\Lambda = k[1 \rightrightarrows 2]$

a) calculate the dimension vectors of all preprojective and all preinjective modules.

b) Show: Any indecomposable which is neither preproj nor pre inj has dimension vector $\begin{bmatrix} n \\ n \end{bmatrix}$ for some n.

$\boxed{1}$ Find an example of a finite dimensional algebra such that $[P_1], \dots, [P_n]$ is <u>not</u> a basis of $K_0(\Lambda)$.

$\boxed{2}$ Calculate the AR quiver of

$$k\left[\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 3 \arrow[r] \arrow[d] & 4 \arrow[r] & 5 \\ & & 6 & & \end{tikzcd}\right].$$

$\boxed{3}$ Calculate Φ for $\Lambda = k[1 \to 2]$ and $\Lambda = k[1 \rightrightarrows 2]$.

What is the order of Φ?

Calculate the Auslander-Reiten quiver of

$\boxed{1}$ $$k\left[\begin{tikzcd} & 2 \arrow[dr, "b"] & \\ 1 \arrow[ur, "a"] & & 3 \arrow[ll, "c"] \end{tikzcd}\right] \Big/ (cba)$$

$\boxed{2}$ $$k\left[\begin{tikzcd} & & 2 \arrow[dr, "b"] & \\ 4 \arrow[r, "d"'] & 1 \arrow[ur, "a"] & & 3 \arrow[ll, "c"] \end{tikzcd}\right] \Big/ (cba)$$

$\boxed{3}$ Assume k is alg. closed.
Classify all local representation finite finite dimensional k-algebras.

Find the Auslander-Reiten quiver of

$\boxed{1}$ $\Lambda = \begin{bmatrix} \mathbb{C} & \mathbb{C} & 0 \\ 0 & \mathbb{R} & 0 \\ 0 & \mathbb{R} & \mathbb{R} \end{bmatrix}$

$\boxed{2}$ $\Lambda = k \left[\begin{tikzcd} & 2 \arrow[dr, "b"] & \\ 1 \arrow[ur, "a"] \arrow[dr, "c"'] & & 4 \arrow[ll, "e"'] \\ & 3 \arrow[ur, "d"'] & \end{tikzcd} \right] \Big/ (eb, ed, ba - dc, bae)$

$\boxed{3}$ Show: If there is an arrow $M \xrightarrow{(a,b)} N$ in the Auslander Reiten quiver with $a \geq 2$ and $b \geq 2$, then there are infinitely many indecomposables.

<u>Hint:</u> Consider length (M), length (N).

1. Find the AR quiver for $k[1 \longrightarrow 2 \longrightarrow 3]$, and all almost split sequences

2. Find the AR quiver for $k\left[\begin{array}{ccc} 1 & \xrightarrow{a} & 2 \\ {\scriptstyle d}\downarrow & & \downarrow{\scriptstyle b} \\ 3 & \xrightarrow[c]{} & 4 \end{array}\right] \Big/ (ba - cd)$, and all almost split sequences.

3. Let $P \overset{p}{\twoheadrightarrow} M$ be an epi from a projective. Show that p is a projective cover if and only if p is right minimal.

$\boxed{1}$ For the (or some) modules found in 1 and 2 last week, find almost split sequences.

$\boxed{2}$ Let $0 \to \tau M \xrightarrow{f} E \xrightarrow{g} M \to 0$ be almost split.
Let $\varphi \in \mathrm{End}(E)$ such that $g\varphi = g$.
Show that φ is an automorphism.

$\boxed{1}$ For $\Lambda = \mathbb{C}\left[\begin{smallmatrix} & & 2 & & \\ & & \downarrow & & \\ 1 & \longrightarrow & 4 & \longleftarrow & 3 \end{smallmatrix}\right]$, find all indecomposable projectives P_i.

Calculate $\tau^- P_i$, $\tau^{-2} P_i, \ldots$.

$\boxed{2}$ For $\Lambda = \mathbb{C}\left[1 \xrightarrow{\alpha} 2 \circlearrowright_{\beta}\right] / (\beta\alpha, \beta^2)$

find all indecomposable projectives P_i.

Calculate $\tau^- P_i$, $\tau^{-2} P_i, \ldots$

$\boxed{3}$ Assume Λ is selfinjective, that is $\operatorname{proj} \Lambda = \operatorname{inj} \Lambda$.

Show that $\tau = \Omega^2 \nu$.

$\boxed{1}$ Finn et moteksempel mot at Nakayama-Lemmaet gjelder for ikke endelig genererte moduler.

$\boxed{2}$ Finn et moteksempel mot at $(\operatorname{Rad} \Lambda) M = \operatorname{Rad} M$ for vilkårlige ringer Λ.

$\boxed{3}$ Finn alle indekomponerbare moduler for $\mathbb{C}[1 \longrightarrow 2 \longrightarrow 3]$.

$\boxed{4}$ Finn alle indekomponerbare moduler for

$$\mathbb{C}\left[\begin{tikzcd} & 2 \arrow[d] & \\ 1 \arrow[r] & 4 & 3 \arrow[l] \end{tikzcd}\right].$$