Comparing algorithms on N datasets. A non-parametric approach

Algorithms may perform totally different on different datasets. Therefore traditional analysis, assuming the same expected value and the error distribution for observations to be normal, is not adequate. Using non-parametric statistics, the assumptions are less strict, and since the analysis often is based on ranks deviations from the assumptions are not that crucial.

Comparing two algorithms on N datasets

Let e_i^1 and e_i^2 be the errors with algorithm 1 and 2 respectively on data set i, $i = 1, 2, \ldots, N$. Using K-fold crossvalidation these may be averages or medians.

N=8

e_i^1	6	7	3	6	9	2	12	13
e_i^2	4	2	5	7	5	4	9	6
$e_i^1 - e_i^2$	2	5	-2	-1	4	-2	3	7

Sign test (Assume continuous symmetric distribution)

Signs: + + - - + - + +

$$X = \sum_{i=1}^{8} X_i \text{ where}$$

$$X_i = \begin{cases} 1 \text{ if algorithm 2 is worse than algoritm 1} \\ 0 \text{ elsewhere} \end{cases}$$

$$H_0: \tilde{\mu}_1 = \tilde{\mu}_2 \qquad H_1: \tilde{\mu}_1 > \tilde{\mu}_2$$

Under H_0: $X \sim B\left(8, \dfrac{1}{2}\right)$. Reject if X is small.

$$P\left(X \leq 3 \middle| p = \dfrac{1}{2}\right) = 0.36 \Rightarrow \text{do not reject.}$$

Wilcoxon signed rank test

$e_i^1 \text{-} e_i^2$	2	5	-2	-1	4	-2	3	7
$\left|e_i^1 - e_i^2\right|$	2	5	2	1	4	2	3	7
Order	1	2	2	2	3	4	5	7
Rank	1	3	3	3	5	6	7	8
Sign	-	-	-	+	+	+	+	+

$W_- = 7.$ With $\alpha = 0.05$, the critical value is 8 (Table A17, WMMY) $\Rightarrow$ reject.

Comparing more than two algorithms on N Datasets

N datasets and k algorithms gives us the datamatrix:
$\left\{X_{ij}\right\}_{N\times k}$. We assume the same error distribution for all
variables. Replace $\left\{X_{ij}\right\}_{N\times k}$ with $\left\{r_{ij}\right\}_{N\times k}$ where r_{ij}, $j=1,\ldots,k$

are the ranks according to algebraic size on data set i. They are
numbers from 1 to k, possibly adjusted for ties. The average
rank for for each i is : $\dfrac{k+1}{2}$.

$$\left\{r_{ij}\right\}_{N\times k} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1k} \\ r_{21} & r_{22} & \cdots & r_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ r_{N1} & r_{N2} & \cdots & r_{Nk} \end{bmatrix}.$$

Compute $\overline{r}_{\cdot j}$, $j=1,\ldots,k$.

Fiedman test (a nonparametric version of the randomized
complete block design analysis).

Use the test statistic

$$Q = \frac{\sum_{j=1}^{k}\left(\bar{r}_{.j} - \frac{k+1}{2}\right)^2}{\frac{\sum_{j=1}^{k}\left(r_{ij} - \frac{k+1}{2}\right)^2}{N(k-1)}} \approx \chi^2(k-1) \text{ if } N > 15 \text{ or } k > 4$$

We have

$$\sum_{j=1}^{k}\left(r_{ij}^2 - r_{ij}(k+1) + \frac{(k+1)}{4}\right) = \frac{k(k+1)(2k+1)}{6} - \frac{k(k+1)^2}{2} + \frac{k(k+1)^2}{4} = \frac{k(k+1)(k-1)}{12}$$

Thereby $Q = \frac{12N}{k(k+1)}\sum_{j=1}^{k}\left(\bar{r}_{.j} - \frac{k+1}{2}\right)^2$.

If Q rejects we can use **Nemenyi's test** to compare two and two algorithms.

The critical distance is given by $CD = \frac{1}{2}q_{k,df,\alpha}\sqrt{\frac{k(k+1)}{6N}}$

where $q_{k,df,\alpha,}$ is the Studentized range statistics.

Studentized Range q Table

The following tables provide the critical value for $q(k, df, \alpha)$ for α = .10, .05, .025, 01, .005 and .001. See [Unplanned Comparisons for ANOVA](#) for more details.

Alpha = 0.10

k -->																			
df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	8.929	13.437	16.358	18.488	20.150	21.504	22.642	23.621	24.477	25.237	25.918	26.536	27.100	27.618	28.097	28.542	28.958	29.347	29.713
2	4.129	5.733	6.772	7.538	8.139	8.633	9.049	9.409	9.725	10.006	10.259	10.488	10.698	10.891	11.070	11.237	11.392	11.538	11.676
3	3.328	4.467	5.199	5.738	6.162	6.511	6.806	7.062	7.287	7.487	7.667	7.831	7.982	8.120	8.248	8.368	8.479	8.584	8.683
4	3.015	3.976	4.586	5.035	5.388	5.679	5.926	6.139	6.327	6.494	6.645	6.783	6.909	7.025	7.132	7.233	7.326	7.414	7.497
5	2.850	3.717	4.264	4.664	4.979	5.238	5.458	5.648	5.816	5.965	6.100	6.223	6.336	6.439	6.536	6.626	6.710	6.788	6.863
6	2.748	3.558	4.065	4.435	4.726	4.966	5.168	5.344	5.499	5.637	5.762	5.875	5.979	6.075	6.164	6.247	6.325	6.398	6.466
7	2.679	3.451	3.931	4.280	4.555	4.780	4.971	5.137	5.283	5.413	5.530	5.637	5.735	5.826	5.910	5.988	6.061	6.130	6.195
8	2.630	3.374	3.834	4.169	4.431	4.646	4.829	4.987	5.126	5.250	5.362	5.464	5.558	5.644	5.724	5.799	5.869	5.935	5.997
9	2.592	3.316	3.761	4.084	4.337	4.545	4.721	4.873	5.007	5.126	5.234	5.333	5.423	5.506	5.583	5.655	5.722	5.786	5.845
10	2.563	3.270	3.704	4.018	4.264	4.465	4.636	4.783	4.913	5.029	5.134	5.229	5.316	5.397	5.472	5.542	5.607	5.668	5.726
11	2.540	3.234	3.658	3.965	4.205	4.401	4.567	4.711	4.838	4.951	5.053	5.145	5.231	5.309	5.382	5.450	5.514	5.573	5.630
12	2.521	3.204	3.621	3.921	4.156	4.349	4.511	4.652	4.776	4.886	4.986	5.076	5.160	5.236	5.308	5.374	5.436	5.495	5.550
13	2.504	3.179	3.589	3.885	4.116	4.304	4.464	4.602	4.724	4.832	4.930	5.019	5.100	5.175	5.245	5.310	5.371	5.429	5.483
14	2.491	3.158	3.563	3.854	4.081	4.267	4.424	4.560	4.679	4.786	4.882	4.969	5.050	5.124	5.192	5.256	5.316	5.372	5.426
15	2.479	3.140	3.540	3.828	4.052	4.235	4.390	4.524	4.641	4.746	4.841	4.927	5.006	5.079	5.146	5.209	5.268	5.324	5.376
16	2.469	3.124	3.520	3.804	4.026	4.207	4.360	4.492	4.608	4.712	4.805	4.890	4.968	5.040	5.106	5.169	5.227	5.282	5.333
17	2.460	3.110	3.503	3.784	4.003	4.182	4.334	4.464	4.579	4.681	4.774	4.857	4.934	5.005	5.071	5.133	5.190	5.244	5.295
18	2.452	3.098	3.487	3.766	3.984	4.161	4.310	4.440	4.553	4.654	4.746	4.829	4.905	4.975	5.040	5.101	5.158	5.211	5.262
19	2.445	3.087	3.474	3.751	3.966	4.142	4.290	4.418	4.530	4.630	4.721	4.803	4.878	4.948	5.012	5.072	5.129	5.182	5.232
20	2.439	3.077	3.462	3.736	3.950	4.124	4.271	4.398	4.510	4.609	4.699	4.780	4.855	4.923	4.987	5.047	5.103	5.155	5.205
21	2.433	3.069	3.451	3.724	3.936	4.109	4.255	4.380	4.491	4.590	4.678	4.759	4.833	4.901	4.965	5.024	5.079	5.131	5.180
22	2.428	3.061	3.441	3.712	3.923	4.095	4.239	4.364	4.474	4.572	4.660	4.740	4.814	4.882	4.944	5.003	5.058	5.109	5.158
23	2.424	3.054	3.432	3.701	3.911	4.082	4.226	4.350	4.459	4.556	4.644	4.723	4.796	4.863	4.926	4.984	5.038	5.089	5.138
24	2.420	3.047	3.423	3.692	3.900	4.070	4.213	4.336	4.445	4.541	4.628	4.707	4.780	4.847	4.909	4.966	5.020	5.071	5.119
25	2.416	3.041	3.416	3.683	3.890	4.059	4.201	4.324	4.432	4.528	4.614	4.693	4.765	4.831	4.893	4.950	5.004	5.055	5.102

df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
26	2.412	3.036	3.409	3.675	3.881	4.049	4.191	4.313	4.420	4.515	4.601	4.680	4.751	4.817	4.878	4.936	4.989	5.039	5.086
27	2.409	3.030	3.402	3.667	3.873	4.040	4.181	4.302	4.409	4.504	4.590	4.667	4.739	4.804	4.865	4.922	4.975	5.025	5.072
28	2.406	3.026	3.396	3.660	3.865	4.032	4.172	4.293	4.399	4.493	4.579	4.656	4.727	4.792	4.853	4.909	4.962	5.012	5.058
29	2.403	3.021	3.391	3.654	3.858	4.024	4.163	4.284	4.389	4.484	4.568	4.645	4.716	4.781	4.841	4.897	4.950	4.999	5.046
30	2.400	3.017	3.386	3.648	3.851	4.016	4.155	4.275	4.381	4.474	4.559	4.635	4.706	4.770	4.830	4.886	4.939	4.988	5.034
31	2.398	3.013	3.381	3.642	3.845	4.009	4.148	4.268	4.372	4.466	4.550	4.626	4.696	4.760	4.820	4.876	4.928	4.977	5.023
32	2.396	3.010	3.376	3.637	3.839	4.003	4.141	4.260	4.365	4.458	4.541	4.617	4.687	4.751	4.811	4.866	4.918	4.967	5.013
33	2.393	3.006	3.372	3.632	3.833	3.997	4.135	4.253	4.357	4.450	4.533	4.609	4.679	4.743	4.802	4.857	4.909	4.957	5.003
34	2.391	3.003	3.368	3.627	3.828	3.991	4.129	4.247	4.351	4.443	4.526	4.602	4.671	4.734	4.794	4.849	4.900	4.949	4.994
35	2.389	3.000	3.364	3.623	3.823	3.986	4.123	4.241	4.344	4.436	4.519	4.594	4.663	4.727	4.786	4.841	4.892	4.940	4.986
36	2.388	2.998	3.361	3.619	3.819	3.981	4.117	4.235	4.338	4.430	4.512	4.588	4.656	4.720	4.778	4.833	4.884	4.932	4.978
37	2.386	2.995	3.357	3.615	3.814	3.976	4.112	4.230	4.332	4.424	4.506	4.581	4.650	4.713	4.771	4.826	4.877	4.925	4.970
38	2.384	2.992	3.354	3.611	3.810	3.972	4.107	4.224	4.327	4.418	4.500	4.575	4.643	4.706	4.765	4.819	4.870	4.918	4.963
39	2.383	2.990	3.351	3.608	3.806	3.967	4.103	4.220	4.322	4.413	4.495	4.569	4.637	4.700	4.758	4.812	4.863	4.911	4.956
40	2.381	2.988	3.348	3.605	3.802	3.963	4.099	4.215	4.317	4.408	4.490	4.564	4.632	4.694	4.752	4.806	4.857	4.904	4.949
48	2.372	2.973	3.330	3.583	3.778	3.937	4.070	4.185	4.285	4.375	4.455	4.528	4.595	4.656	4.713	4.766	4.816	4.863	4.907
60	2.363	2.959	3.312	3.562	3.755	3.911	4.042	4.155	4.254	4.342	4.421	4.493	4.558	4.619	4.675	4.727	4.775	4.821	4.864
80	2.353	2.945	3.294	3.541	3.731	3.885	4.014	4.125	4.223	4.309	4.387	4.457	4.521	4.581	4.636	4.687	4.735	4.780	4.822
120	2.344	2.930	3.276	3.520	3.707	3.859	3.986	4.096	4.191	4.276	4.353	4.422	4.485	4.543	4.597	4.647	4.694	4.738	4.779
240	2.335	2.916	3.258	3.499	3.684	3.834	3.959	4.066	4.160	4.244	4.319	4.386	4.448	4.505	4.558	4.607	4.653	4.696	4.737
Inf	2.326	2.902	3.240	3.478	3.661	3.808	3.931	4.037	4.129	4.211	4.285	4.351	4.412	4.468	4.519	4.568	4.612	4.654	4.694

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	29.71	30.39	30.99	31.54	32.04	32.5	32.93	33.33	33.71	34.06	34.38	35.79	36.91	37.83	38.62	39.3	39.91
2	11.68	11.93	12.16	12.36	12.55	12.73	12.89	13.04	13.18	13.31	13.44	13.97	14.40	14.75	15.05	15.31	15.54
3	8.683	8.864	9.029	9.177	9.314	9.440	9.557	9.666	9.768	9.864	9.954	10.34	10.65	10.91	11.12	11.31	11.48
4	7.497	7.650	7.789	7.914	8.029	8.135	8.234	8.326	8.412	8.493	8.569	8.806	9.156	9.373	9.557	9.718	9.860
5	6.863	7.000	7.123	7.236	7.340	7.435	7.523	7.606	7.683	7.756	7.825	8.118	8.353	8.548	8.715	8.859	8.988
6	6.466	6.593	6.708	6.812	6.908	6.996	7.078	7.155	7.227	7.294	7.358	7.630	7.848	8.029	8.184	8.319	8.438
7	6.195	6.315	6.422	6.521	6.611	6.695	6.773	6.845	6.913	6.976	7.036	7.294	7.500	7.672	7.818	7.946	8.059
8	5.997	6.111	6.214	6.308	6.395	6.475	6.549	6.618	6.683	6.744	6.801	7.048	7.245	7.409	7.550	7.672	7.780
9	5.845	5.956	6.055	6.146	6.229	6.306	6.378	6.444	6.507	6.566	6.621	6.859	7.050	7.208	7.343	7.461	7.566
10	5.726	5.833	5.930	6.017	6.098	6.173	6.242	6.307	6.368	6.425	6.479	6.709	6.895	7.048	7.180	7.295	7.396
11	5.630	5.734	5.828	5.914	5.992	6.065	6.132	6.196	6.255	6.310	6.363	6.588	6.768	6.918	7.047	7.158	7.258
12	5.550	5.652	5.744	5.827	5.904	5.976	6.042	6.103	6.161	6.215	6.267	6.487	6.663	6.810	6.936	7.045	7.142
13	5.483	5.583	5.673	5.755	5.830	5.900	5.965	6.025	6.082	6.135	6.186	6.402	6.575	6.719	6.842	6.949	7.045
14	5.426	5.524	5.612	5.693	5.767	5.836	5.899	5.959	6.014	6.067	6.116	6.329	6.499	6.641	6.762	6.868	6.961
15	5.376	5.473	5.560	5.639	5.713	5.780	5.843	5.901	5.956	6.008	6.057	6.266	6.433	6.573	6.692	6.796	6.888
16	5.333	5.428	5.515	5.593	5.665	5.732	5.793	5.851	5.905	5.956	6.004	6.210	6.376	6.513	6.631	6.734	6.825
17	5.295	5.389	5.474	5.552	5.623	5.689	5.750	5.806	5.860	5.910	5.958	6.162	6.325	6.461	6.577	6.679	6.769
18	5.262	5.355	5.439	5.515	5.585	5.650	5.711	5.767	5.820	5.870	5.917	6.113	6.280	6.414	6.529	6.630	6.719
19	5.232	5.324	5.407	5.483	5.552	5.616	5.676	5.732	5.784	5.833	5.880	6.079	6.239	6.372	6.486	6.585	6.674
20	5.205	5.296	5.378	5.453	5.522	5.586	5.645	5.700	5.752	5.801	5.847	6.044	6.203	6.335	6.447	6.546	6.633
24	5.119	5.208	5.287	5.360	5.427	5.489	5.546	5.600	5.650	5.697	5.741	5.933	6.086	6.214	6.324	6.419	6.503
30	5.034	5.120	5.197	5.267	5.332	5.392	5.447	5.499	5.547	5.593	5.636	5.821	5.969	6.093	6.198	6.291	6.372
40	4.949	5.032	5.107	5.174	5.236	5.294	5.347	5.397	5.444	5.488	5.529	5.708	5.850	5.969	6.071	6.160	6.238
60	4.864	4.944	5.015	5.081	5.141	5.196	5.247	5.295	5.340	5.382	5.422	5.593	5.730	5.844	5.941	6.026	6.102
120	4.779	4.656	4.924	4.987	5.044	5.097	5.146	5.192	5.235	5.275	5.313	5.476	5.606	5.715	5.808	5.888	5.960
inf	4.694	4.767	4.832	4.892	4.947	4.997	5.044	5.087	5.128	5.166	5.202	5.357	5.480	5.582	5.669	5.745	5.812

Alpha = 0.05

df	k --> 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17.969	26.976	32.819	37.082	40.408	43.119	45.397	47.357	49.071	50.592	51.957	53.194	54.323	55.361	56.320	57.212	58.044	58.824	59.558
2	6.085	8.331	9.798	10.881	11.734	12.435	13.027	13.539	13.988	14.389	14.749	15.076	15.375	15.650	15.905	16.143	16.365	16.573	16.769
3	4.501	5.910	6.825	7.502	8.037	8.478	8.852	9.177	9.462	9.717	9.946	10.155	10.346	10.522	10.686	10.838	10.980	11.114	11.240
4	3.926	5.040	5.757	6.287	6.706	7.053	7.347	7.602	7.826	8.027	8.208	8.373	8.524	8.664	8.793	8.914	9.027	9.133	9.233
5	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.801	6.995	7.167	7.323	7.466	7.596	7.716	7.828	7.932	8.030	8.122	8.208
6	3.460	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493	6.649	6.789	6.917	7.034	7.143	7.244	7.338	7.426	7.508	7.586
7	3.344	4.165	4.681	5.060	5.359	5.606	5.815	5.997	6.158	6.302	6.431	6.550	6.658	6.759	6.852	6.939	7.020	7.097	7.169
8	3.261	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918	6.053	6.175	6.287	6.389	6.483	6.571	6.653	6.729	6.801	6.869
9	3.199	3.948	4.415	4.755	5.024	5.244	5.432	5.595	5.738	5.867	5.983	6.089	6.186	6.276	6.359	6.437	6.510	6.579	6.643
10	3.151	3.877	4.327	4.654	4.912	5.124	5.304	5.460	5.598	5.722	5.833	5.935	6.028	6.114	6.194	6.269	6.339	6.405	6.467
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.486	5.605	5.713	5.811	5.901	5.984	6.062	6.134	6.202	6.265	6.325
12	3.081	3.773	4.199	4.508	4.750	4.950	5.119	5.265	5.395	5.510	5.615	5.710	5.797	5.878	5.953	6.023	6.089	6.151	6.209
13	3.055	3.734	4.151	4.453	4.690	4.884	5.049	5.192	5.318	5.431	5.533	5.625	5.711	5.789	5.862	5.931	5.995	6.055	6.112
14	3.033	3.701	4.111	4.407	4.639	4.829	4.990	5.130	5.253	5.364	5.463	5.554	5.637	5.714	5.785	5.852	5.915	5.973	6.029
15	3.014	3.673	4.076	4.367	4.595	4.782	4.940	5.077	5.198	5.306	5.403	5.492	5.574	5.649	5.719	5.785	5.846	5.904	5.958
16	2.998	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.150	5.256	5.352	5.439	5.519	5.593	5.662	5.726	5.786	5.843	5.896
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108	5.212	5.306	5.392	5.471	5.544	5.612	5.675	5.734	5.790	5.842
18	2.971	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071	5.173	5.266	5.351	5.429	5.501	5.567	5.629	5.688	5.743	5.794
19	2.960	3.593	3.977	4.253	4.468	4.645	4.794	4.924	5.037	5.139	5.231	5.314	5.391	5.462	5.528	5.589	5.647	5.701	5.752
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.895	5.008	5.108	5.199	5.282	5.357	5.427	5.492	5.553	5.610	5.663	5.714
21	2.941	3.565	3.942	4.213	4.424	4.597	4.743	4.870	4.981	5.081	5.170	5.252	5.327	5.396	5.460	5.520	5.576	5.629	5.679
22	2.933	3.553	3.927	4.196	4.405	4.577	4.722	4.847	4.957	5.056	5.144	5.225	5.299	5.368	5.431	5.491	5.546	5.599	5.648
23	2.926	3.542	3.914	4.180	4.388	4.558	4.702	4.826	4.935	5.033	5.121	5.201	5.274	5.342	5.405	5.464	5.519	5.571	5.620
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915	5.012	5.099	5.179	5.251	5.319	5.381	5.439	5.494	5.545	5.594
25	2.913	3.523	3.890	4.153	4.358	4.526	4.667	4.789	4.897	4.993	5.079	5.158	5.230	5.297	5.359	5.417	5.471	5.522	5.570

df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
26	2.907	3.514	3.880	4.141	4.345	4.511	4.652	4.773	4.880	4.975	5.061	5.139	5.211	5.277	5.339	5.396	5.450	5.500	5.548
27	2.902	3.506	3.870	4.130	4.333	4.498	4.638	4.758	4.864	4.959	5.044	5.122	5.193	5.259	5.320	5.377	5.430	5.480	5.528
28	2.897	3.499	3.861	4.120	4.322	4.486	4.625	4.745	4.850	4.944	5.029	5.106	5.177	5.242	5.302	5.359	5.412	5.462	5.509
29	2.892	3.493	3.853	4.111	4.311	4.475	4.613	4.732	4.837	4.930	5.014	5.091	5.161	5.226	5.286	5.342	5.395	5.445	5.491
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.720	4.824	4.917	5.001	5.077	5.147	5.211	5.271	5.327	5.379	5.429	5.475
31	2.884	3.481	3.838	4.094	4.292	4.454	4.591	4.709	4.812	4.905	4.988	5.064	5.134	5.198	5.257	5.313	5.365	5.414	5.460
32	2.881	3.475	3.832	4.086	4.284	4.445	4.581	4.698	4.802	4.894	4.976	5.052	5.121	5.185	5.244	5.299	5.351	5.400	5.445
33	2.877	3.470	3.825	4.079	4.276	4.436	4.572	4.689	4.791	4.883	4.965	5.040	5.109	5.173	5.232	5.287	5.338	5.386	5.432
34	2.874	3.465	3.820	4.072	4.268	4.428	4.563	4.680	4.782	4.873	4.955	5.030	5.098	5.161	5.220	5.275	5.326	5.374	5.420
35	2.871	3.461	3.814	4.066	4.261	4.421	4.555	4.671	4.773	4.863	4.945	5.020	5.088	5.151	5.209	5.264	5.315	5.362	5.408
36	2.868	3.457	3.809	4.060	4.255	4.414	4.547	4.663	4.764	4.855	4.936	5.010	5.078	5.141	5.199	5.253	5.304	5.352	5.397
37	2.865	3.453	3.804	4.054	4.249	4.407	4.540	4.655	4.756	4.846	4.927	5.001	5.069	5.131	5.189	5.243	5.294	5.341	5.386
38	2.863	3.449	3.799	4.049	4.243	4.400	4.533	4.648	4.749	4.838	4.919	4.993	5.060	5.122	5.180	5.234	5.284	5.331	5.376
39	2.861	3.445	3.795	4.044	4.237	4.394	4.527	4.641	4.741	4.831	4.911	4.985	5.052	5.114	5.171	5.225	5.275	5.322	5.367
40	2.858	3.442	3.791	4.039	4.232	4.388	4.521	4.634	4.735	4.824	4.904	4.977	5.044	5.106	5.163	5.216	5.266	5.313	5.358
48	2.843	3.420	3.764	4.008	4.197	4.351	4.481	4.592	4.690	4.777	4.856	4.927	4.993	5.053	5.109	5.161	5.210	5.256	5.299
60	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646	4.732	4.808	4.878	4.942	5.001	5.056	5.107	5.154	5.199	5.241
80	2.814	3.377	3.711	3.947	4.129	4.277	4.402	4.509	4.603	4.686	4.761	4.829	4.892	4.949	5.003	5.052	5.099	5.142	5.183
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560	4.641	4.714	4.781	4.842	4.898	4.950	4.998	5.043	5.086	5.126
240	2.786	3.335	3.659	3.887	4.063	4.205	4.324	4.427	4.517	4.596	4.668	4.733	4.792	4.847	4.897	4.944	4.988	5.030	5.069
inf	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474	4.552	4.622	4.685	4.743	4.796	4.845	4.891	4.934	4.974	5.012

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	59.56	60.91	62.12	63.22	64.23	65.15	66.01	66.81	67.56	68.26	68.92	71.73	73.97	75.82	77.4	78.77	79.98
2	16.77	17.13	17.45	17.75	18.02	18.27	18.5	18.72	18.92	19.11	19.28	20.05	20.66	21.16	21.59	21.96	22.29
3	11.24	11.47	11.68	11.87	12.05	12.21	12.36	12.50	12.63	12.75	12.87	13.36	13.76	14.08	14.36	14.61	14.82
4	9.233	9.418	9.584	9.736	9.875	10.00	10.12	10.23	10.34	10.44	10.53	10.93	11.24	11.51	11.73	11.92	12.09
5	8.208	8.368	8.512	8.643	8.764	8.875	8.979	9.075	9.165	9.250	9.330	9.674	9.949	10.18	10.38	10.54	10.69
6	7.587	7.730	7.861	7.979	8.088	8.189	8.283	8.370	8.452	8.529	8.601	8.913	9.163	9.370	9.548	9.702	9.839
7	7.170	7.303	7.423	7.533	7.634	7.728	7.814	7.895	7.972	8.043	8.110	8.400	8.632	8.824	8.989	9.133	9.261
8	6.870	6.995	7.109	7.212	7.307	7.395	7.477	7.554	7.625	7.693	7.756	8.029	8.248	8.430	8.586	8.722	8.843
9	6.644	6.763	6.871	6.970	7.061	7.145	7.222	7.295	7.363	7.428	7.488	7.749	7.958	8.132	8.281	8.410	8.526
10	6.467	6.582	6.686	6.781	6.868	6.948	7.023	7.093	7.159	7.220	7.279	7.529	7.730	7.897	8.041	8.166	8.276
11	6.326	6.436	6.536	6.628	6.712	6.790	6.863	6.930	6.994	7.053	7.110	7.352	7.546	7.708	7.847	7.968	8.075
12	6.209	6.317	6.414	6.503	6.585	6.660	6.731	6.796	6.858	6.916	6.970	7.205	7.394	7.552	7.687	7.804	7.909
13	6.112	6.217	6.312	6.398	6.478	6.551	6.620	6.684	6.744	6.800	6.854	7.083	7.267	7.421	7.552	7.667	7.769
14	6.029	6.132	6.224	6.309	6.387	6.459	6.526	6.588	6.647	6.702	6.754	6.979	7.159	7.309	7.438	7.550	7.650
15	5.958	6.059	6.149	6.233	6.309	6.379	6.445	6.506	6.564	6.618	6.669	6.888	7.065	7.212	7.339	7.449	7.546
16	5.897	5.995	6.084	6.166	6.241	6.310	6.374	6.434	6.491	6.544	6.594	6.810	6.984	7.128	7.252	7.360	7.457
17	5.842	5.940	6.027	6.107	6.181	6.249	6.313	6.372	6.427	6.479	6.529	6.741	6.912	7.054	7.176	7.283	7.377
18	5.794	5.890	5.977	6.055	6.128	6.195	6.258	6.316	6.371	6.422	6.471	6.680	6.848	6.989	7.109	7.213	7.307
19	5.752	5.846	5.932	6.009	6.081	6.147	6.209	6.267	6.321	6.371	6.419	6.626	6.792	6.930	7.048	7.152	7.244
20	5.714	5.807	5.891	5.968	6.039	6.104	6.165	6.222	6.275	6.325	6.373	6.576	6.740	6.877	6.994	7.097	7.187
24	5.594	5.683	5.764	5.838	5.906	5.968	6.027	6.081	6.132	6.181	6.226	6.421	6.579	6.710	6.822	6.920	7.008
30	5.475	5.561	5.638	5.709	5.774	5.833	5.889	5.941	5.990	6.037	6.080	6.267	6.417	6.543	6.650	6.744	6.827
40	5.358	5.439	5.513	5.581	5.642	5.700	5.753	5.803	5.849	5.893	5.934	6.112	6.255	6.375	6.477	6.566	6.645
60	5.241	5.319	5.389	5.453	5.512	5.566	5.617	5.664	5.708	5.750	5.789	5.958	6.093	6.206	6.303	6.387	6.462
120	5.126	5.200	5.266	5.327	5.382	5.434	5.481	5.526	5.568	5.607	5.644	5.802	5.929	6.035	6.126	6.205	6.275
inf	5.012	5.081	5.144	5.201	5.253	5.301	5.346	5.388	5.427	5.463	5.498	5.646	5.764	5.863	5.947	6.020	6.085

Alpha = 0.025

df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20
1	35.99	54.00	65.69	74.22	80.87	86.29	90.85	94.77	98.20	101.3	104.0	106.5	108.8	110.8	112.7	116.2	119.2
2	8.776	11.94	14.01	15.54	16.75	17.74	18.58	19.31	19.95	20.52	21.03	21.49	21.91	22.30	22.67	23.32	23.89
3	5.907	7.661	8.808	9.660	10.34	10.89	11.37	11.78	12.14	12.46	12.75	13.01	13.26	13.48	13.69	14.06	14.39
4	4.943	6.244	7.088	7.716	8.213	8.625	8.976	9.279	9.548	9.788	10.01	10.20	10.39	10.55	10.71	10.99	11.23
5	4.474	5.558	6.257	6.775	7.186	7.527	7.816	8.068	8.291	8.490	8.670	8.834	8.984	9.124	9.253	9.486	9.693
6	4.199	5.158	5.772	6.226	6.586	6.884	7.138	7.359	7.554	7.729	7.887	8.031	8.163	8.286	8.399	8.605	8.787
7	4.018	4.897	5.455	5.868	6.194	6.464	6.695	6.895	7.072	7.230	7.373	7.504	7.624	7.735	7.839	8.025	8.191
8	3.892	4.714	5.233	5.616	5.919	6.169	6.382	6.568	6.732	6.879	7.011	7.132	7.244	7.347	7.443	7.616	7.769
9	3.797	4.578	5.069	5.430	5.715	5.950	6.151	6.325	6.479	6.617	6.742	6.856	6.961	7.058	7.148	7.311	7.455
10	3.725	4.474	4.943	5.287	5.558	5.782	5.972	6.138	6.285	6.416	6.534	6.643	6.742	6.834	6.920	7.075	7.212
11	3.667	4.391	4.843	5.173	5.433	5.648	5.831	5.989	6.130	6.256	6.369	6.473	6.568	6.657	6.739	6.887	7.019
12	3.620	4.325	4.762	5.081	5.332	5.540	5.716	5.869	6.004	6.125	6.235	6.335	6.427	6.512	6.591	6.734	6.861
13	3.582	4.269	4.694	5.004	5.248	5.449	5.620	5.769	5.900	6.017	6.123	6.220	6.309	6.392	6.468	6.607	6.730
14	3.550	4.222	4.638	4.940	5.178	5.374	5.540	5.684	5.811	5.926	6.029	6.123	6.210	6.290	6.364	6.499	6.619
15	3.522	4.182	4.589	4.885	5.118	5.309	5.471	5.612	5.737	5.848	5.949	6.041	6.125	6.203	6.276	6.407	6.523
16	3.498	4.148	4.548	4.838	5.066	5.253	5.412	5.550	5.672	5.781	5.879	5.969	6.052	6.128	6.199	6.328	6.441
17	3.477	4.118	4.512	4.797	5.020	5.204	5.361	5.496	5.615	5.722	5.818	5.907	5.987	6.062	6.132	6.258	6.370
18	3.458	4.092	4.480	4.761	4.981	5.162	5.315	5.448	5.565	5.670	5.765	5.852	5.931	6.004	6.073	6.197	6.306
19	3.442	4.068	4.451	4.728	4.945	5.123	5.275	5.405	5.521	5.624	5.718	5.803	5.881	5.954	6.020	6.142	6.250
20	3.427	4.047	4.426	4.700	4.914	5.089	5.238	5.368	5.481	5.583	5.675	5.759	5.836	5.907	5.974	6.093	6.200
24	3.381	3.983	4.347	4.610	4.816	4.984	5.216	5.250	5.358	5.455	5.543	5.623	5.697	5.764	5.827	5.941	6.043
30	3.337	3.919	4.271	4.523	4.720	4.881	5.017	5.134	5.238	5.330	5.414	5.490	5.560	5.624	5.684	5.792	5.888
40	3.294	3.858	4.197	4.439	4.627	4.780	4.910	5.022	5.120	5.208	5.288	5.360	5.426	5.487	5.544	5.646	5.737
60	3.251	3.798	4.124	4.356	4.536	4.682	4.806	4.912	5.006	5.089	5.164	5.232	5.295	5.352	5.406	5.503	5.588
120	3.210	3.739	4.053	4.276	4.447	4.587	4.704	4.805	4.894	4.972	5.043	5.107	5.166	5.221	5.271	5.362	5.442
inf	3.170	3.682	3.984	4.197	4.361	4.494	4.605	4.700	4.784	4.858	4.925	4.985	5.041	5.092	5.139	5.224	5.299

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	119.2	121.9	124.3	126.5	128.6	130.4	132.1	133.7	135.2	136.6	137.9	143.6	148.1	151.8	154.9	157.7	160.0
2	23.89	24.41	24.87	25.29	25.67	26.03	26.35	26.66	26.95	27.22	27.47	28.55	29.42	30.13	30.74	31.27	31.74
3	14.39	14.69	14.95	15.19	15.41	15.62	15.81	15.99	16.15	16.31	16.46	17.08	17.59	18.00	18.36	18.67	18.95
4	11.23	11.46	11.66	11.84	12.00	12.16	12.30	12.44	12.56	12.68	12.79	13.27	13.65	13.96	14.23	14.47	14.68
5	9.693	9.878	10.04	10.20	10.34	10.47	10.59	10.70	10.80	10.91	11.00	11.40	11.72	11.99	12.21	12.41	12.59
6	8.787	8.949	9.097	9.231	9.355	9.469	9.575	9.674	9.767	9.855	9.938	10.30	10.58	10.81	11.02	11.19	11.35
7	8.191	8.339	8.473	8.595	8.708	8.812	8.909	8.999	9.084	9.164	9.239	9.563	9.822	10.04	10.23	10.38	10.53
8	7.769	7.907	8.031	8.145	8.250	8.346	8.436	8.520	8.599	8.673	8.743	9.044	9.286	9.487	9.660	9.810	9.944
9	7.455	7.585	7.702	7.809	7.908	7.999	8.084	8.163	8.237	8.307	8.373	8.657	8.885	9.076	9.238	9.381	9.507
10	7.212	7.335	7.447	7.549	7.643	7.729	7.810	7.885	7.956	8.023	8.086	8.356	8.574	8.755	8.911	9.046	9.167
11	7.019	7.137	7.244	7.341	7.431	7.514	7.592	7.664	7.732	7.796	7.856	8.116	8.325	8.499	8.648	8.779	8.894
12	6.861	6.974	7.078	7.172	7.258	7.338	7.413	7.483	7.548	7.610	7.668	7.919	8.120	8.289	8.433	8.559	8.671
13	6.730	6.840	6.939	7.031	7.115	7.192	7.265	7.332	7.396	7.455	7.512	7.755	7.950	8.113	8.253	8.375	8.484
14	6.619	6.726	6.823	6.911	6.993	7.069	7.139	7.204	7.266	7.324	7.379	7.615	7.806	7.965	8.101	8.220	8.325
15	6.523	6.628	6.723	6.809	6.889	6.962	7.031	7.095	7.155	7.212	7.265	7.496	7.682	7.837	7.970	8.086	8.189
16	6.441	6.543	6.636	6.721	6.799	6.870	6.938	7.000	7.059	7.115	7.167	7.393	7.574	7.726	7.856	7.969	8.070
17	6.370	6.469	6.560	6.644	6.720	6.790	6.856	6.917	6.975	7.030	7.081	7.302	7.480	7.628	7.756	7.868	7.966
18	6.306	6.404	6.493	6.575	6.650	6.720	6.784	6.844	6.900	6.954	7.005	7.221	7.396	7.543	7.667	7.777	7.874
19	6.250	6.347	6.434	6.514	6.588	6.656	6.719	6.779	6.835	6.887	6.936	7.150	7.322	7.465	7.589	7.696	7.792
20	6.200	6.295	6.381	6.460	6.532	6.600	6.662	6.720	6.775	6.827	6.876	7.086	7.255	7.397	7.518	7.624	7.718
24	6.043	6.133	6.215	6.290	6.359	6.423	6.482	6.538	6.589	6.639	6.685	6.885	7.046	7.180	7.296	7.397	7.486
30	5.888	5.974	6.052	6.123	6.188	6.248	6.305	6.357	6.406	6.453	6.497	6.686	6.839	6.965	7.075	7.171	7.255
40	5.737	5.818	5.891	5.958	6.020	6.077	6.130	6.179	6.226	6.270	6.311	6.489	6.633	6.753	6.855	6.945	7.025
60	5.588	5.664	5.733	5.797	5.854	5.908	5.958	6.004	6.048	6.089	6.127	6.295	6.429	6.540	6.636	6.720	6.795
120	5.442	5.513	5.578	5.637	5.691	5.741	5.788	5.831	5.872	5.910	5.946	6.101	6.225	6.329	6.418	6.495	6.564
inf	5.299	5.365	5.425	5.480	5.530	5.577	5.620	5.660	5.698	5.733	5.766	5.909	6.023	6.118	6.199	6.270	6.333

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df	k --> 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	90.024	135.04	164.26	185.58	202.21	215.77	227.17	236.97	245.54	253.15	259.98	266.17	271.81	277.00	281.80	286.26	290.43	294.33	298.00
2	14.036	19.019	22.294	24.717	26.629	28.201	29.530	30.679	31.689	32.589	33.398	34.134	34.806	35.426	36.000	36.534	37.034	37.502	37.943
3	8.260	10.619	12.170	13.324	14.241	14.998	15.641	16.199	16.691	17.130	17.526	17.887	18.217	18.522	18.805	19.068	19.315	19.546	19.765
4	6.511	8.120	9.173	9.958	10.583	11.101	11.542	11.925	12.264	12.567	12.840	13.090	13.318	13.530	13.726	13.909	14.081	14.242	14.394
5	5.702	6.976	7.804	8.421	8.913	9.321	9.669	9.971	10.239	10.479	10.696	10.894	11.076	11.244	11.400	11.545	11.682	11.811	11.932
6	5.243	6.331	7.033	7.556	7.972	8.318	8.612	8.869	9.097	9.300	9.485	9.653	9.808	9.951	10.084	10.208	10.325	10.434	10.538
7	4.949	5.919	6.542	7.005	7.373	7.678	7.939	8.166	8.367	8.548	8.711	8.860	8.997	9.124	9.242	9.353	9.456	9.553	9.645
8	4.745	5.635	6.204	6.625	6.959	7.237	7.474	7.680	7.863	8.027	8.176	8.311	8.436	8.552	8.659	8.760	8.854	8.943	9.027
9	4.596	5.428	5.957	6.347	6.657	6.915	7.134	7.325	7.494	7.646	7.784	7.910	8.025	8.132	8.232	8.325	8.412	8.495	8.573
10	4.482	5.270	5.769	6.136	6.428	6.669	6.875	7.054	7.213	7.356	7.485	7.603	7.712	7.812	7.906	7.993	8.075	8.153	8.226
11	4.392	5.146	5.621	5.970	6.247	6.476	6.671	6.841	6.992	7.127	7.250	7.362	7.464	7.560	7.648	7.731	7.809	7.883	7.952
12	4.320	5.046	5.502	5.836	6.101	6.320	6.507	6.670	6.814	6.943	7.060	7.166	7.265	7.356	7.441	7.520	7.594	7.664	7.730
13	4.260	4.964	5.404	5.726	5.981	6.192	6.372	6.528	6.666	6.791	6.903	7.006	7.100	7.188	7.269	7.345	7.417	7.484	7.548
14	4.210	4.895	5.322	5.634	5.881	6.085	6.258	6.409	6.543	6.663	6.772	6.871	6.962	7.047	7.125	7.199	7.268	7.333	7.394
15	4.167	4.836	5.252	5.556	5.796	5.994	6.162	6.309	6.438	6.555	6.660	6.756	6.845	6.927	7.003	7.074	7.141	7.204	7.264
16	4.131	4.786	5.192	5.489	5.722	5.915	6.079	6.222	6.348	6.461	6.564	6.658	6.744	6.823	6.897	6.967	7.032	7.093	7.151
17	4.099	4.742	5.140	5.430	5.659	5.847	6.007	6.147	6.270	6.380	6.480	6.572	6.656	6.733	6.806	6.873	6.937	6.997	7.053
18	4.071	4.703	5.094	5.379	5.603	5.787	5.944	6.081	6.201	6.309	6.407	6.496	6.579	6.655	6.725	6.791	6.854	6.912	6.967
19	4.046	4.669	5.054	5.334	5.553	5.735	5.889	6.022	6.141	6.246	6.342	6.430	6.510	6.585	6.654	6.719	6.780	6.837	6.891
20	4.024	4.639	5.018	5.293	5.510	5.688	5.839	5.970	6.086	6.190	6.285	6.370	6.449	6.523	6.591	6.654	6.714	6.770	6.823
21	4.004	4.612	4.986	5.257	5.470	5.646	5.794	5.924	6.038	6.140	6.233	6.317	6.395	6.467	6.534	6.596	6.655	6.710	6.762
22	3.986	4.588	4.957	5.225	5.435	5.608	5.754	5.882	5.994	6.095	6.186	6.269	6.346	6.417	6.482	6.544	6.602	6.656	6.707
23	3.970	4.566	4.931	5.195	5.403	5.573	5.718	5.844	5.955	6.054	6.144	6.226	6.301	6.371	6.436	6.497	6.553	6.607	6.658
24	3.955	4.546	4.907	5.168	5.373	5.542	5.685	5.809	5.919	6.017	6.105	6.186	6.261	6.330	6.394	6.453	6.510	6.562	6.612
25	3.942	4.527	4.885	5.144	5.347	5.513	5.655	5.778	5.886	5.983	6.070	6.150	6.224	6.292	6.355	6.414	6.469	6.522	6.571

df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
26	3.930	4.510	4.865	5.121	5.322	5.487	5.627	5.749	5.856	5.951	6.038	6.117	6.190	6.257	6.319	6.378	6.432	6.484	6.533
27	3.918	4.495	4.847	5.101	5.300	5.463	5.602	5.722	5.828	5.923	6.008	6.087	6.158	6.225	6.287	6.344	6.399	6.450	6.498
28	3.908	4.481	4.830	5.082	5.279	5.441	5.578	5.697	5.802	5.896	5.981	6.058	6.129	6.195	6.256	6.314	6.367	6.418	6.465
29	3.898	4.467	4.814	5.064	5.260	5.420	5.556	5.674	5.778	5.871	5.955	6.032	6.103	6.168	6.228	6.285	6.338	6.388	6.435
30	3.889	4.455	4.799	5.048	5.242	5.401	5.536	5.653	5.756	5.848	5.932	6.008	6.078	6.142	6.202	6.258	6.311	6.361	6.407
31	3.881	4.443	4.786	5.032	5.225	5.383	5.517	5.633	5.736	5.827	5.910	5.985	6.055	6.119	6.178	6.234	6.286	6.335	6.381
32	3.873	4.433	4.773	5.018	5.210	5.367	5.500	5.615	5.716	5.807	5.889	5.964	6.033	6.096	6.155	6.211	6.262	6.311	6.357
33	3.865	4.423	4.761	5.005	5.195	5.351	5.483	5.598	5.698	5.789	5.870	5.944	6.013	6.076	6.134	6.189	6.240	6.289	6.334
34	3.859	4.413	4.750	4.992	5.181	5.336	5.468	5.581	5.682	5.771	5.852	5.926	5.994	6.056	6.114	6.169	6.220	6.268	6.313
35	3.852	4.404	4.739	4.980	5.169	5.323	5.453	5.566	5.666	5.755	5.835	5.908	5.976	6.038	6.096	6.150	6.200	6.248	6.293
36	3.846	4.396	4.729	4.969	5.156	5.310	5.439	5.552	5.651	5.739	5.819	5.892	5.959	6.021	6.078	6.132	6.182	6.229	6.274
37	3.840	4.388	4.720	4.959	5.145	5.298	5.427	5.538	5.637	5.725	5.804	5.876	5.943	6.004	6.061	6.115	6.165	6.212	6.256
38	3.835	4.381	4.711	4.949	5.134	5.286	5.414	5.526	5.623	5.711	5.790	5.862	5.928	5.989	6.046	6.099	6.148	6.195	6.239
39	3.830	4.374	4.703	4.940	5.124	5.275	5.403	5.513	5.611	5.698	5.776	5.848	5.914	5.974	6.031	6.084	6.133	6.179	6.223
40	3.825	4.367	4.695	4.931	5.114	5.265	5.392	5.502	5.599	5.685	5.764	5.835	5.900	5.961	6.017	6.069	6.118	6.165	6.208
48	3.793	4.324	4.644	4.874	5.052	5.198	5.322	5.428	5.522	5.606	5.681	5.750	5.814	5.872	5.926	5.977	6.024	6.069	6.111
60	3.762	4.282	4.594	4.818	4.991	5.133	5.253	5.356	5.447	5.528	5.601	5.667	5.728	5.784	5.837	5.886	5.931	5.974	6.015
80	3.732	4.241	4.545	4.763	4.931	5.069	5.185	5.284	5.372	5.451	5.521	5.585	5.644	5.698	5.749	5.796	5.840	5.881	5.920
120	3.702	4.200	4.497	4.709	4.872	5.005	5.118	5.214	5.299	5.375	5.443	5.505	5.561	5.614	5.662	5.708	5.750	5.790	5.827
240	3.672	4.160	4.450	4.655	4.814	4.943	5.052	5.145	5.227	5.300	5.366	5.426	5.480	5.530	5.577	5.621	5.661	5.699	5.735
inf	3.643	4.120	4.403	4.603	4.757	4.882	4.987	5.078	5.157	5.227	5.290	5.348	5.400	5.448	5.493	5.535	5.574	5.611	5.645

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	298.0	304.7	310.8	316.3	321.3	326.0	330.3	334.3	338.0	341.5	344.8	358.9	370.1	379.4	387.3	394.1	400.1
2	37.95	38.76	39.49	40.15	40.76	41.32	41.84	42.33	42.78	43.21	43.61	45.33	46.70	47.83	48.80	49.64	50.38
3	19.77	20.17	20.53	20.86	21.16	21.44	21.70	21.95	22.17	22.39	22.59	23.45	24.13	24.71	25.19	25.67	25.99
4	14.40	14.68	14.93	15.16	15.37	15.57	15.75	15.92	16.08	16.23	16.37	16.98	17.46	17.86	18.20	18.50	18.77
5	11.93	12.16	12.36	12.54	12.71	12.87	13.02	13.15	13.28	13.40	13.52	14.00	14.39	14.72	14.99	15.23	15.45
6	10.54	10.73	10.91	11.06	11.21	11.34	11.47	11.58	11.69	11.80	11.90	12.31	12.65	12.92	13.16	13.37	13.55
7	9.646	9.815	9.970	10.11	10.24	10.36	10.47	10.58	10.67	10.77	10.85	11.23	11.52	11.77	11.99	12.17	12.34
8	9.027	9.182	9.322	9.450	9.569	9.678	9.779	9.874	9.964	10.05	10.13	10.47	10.75	10.97	11.17	11.34	11.49
9	8.573	8.717	8.847	8.966	9.075	9.177	9.271	9.360	9.443	9.521	9.594	9.912	10.17	10.38	10.57	10.73	10.87
10	8.226	8.361	8.483	8.595	8.698	8.794	8.883	8.966	9.044	9.117	9.187	9.486	9.726	9.927	10.10	10.25	10.39
11	7.952	8.080	8.196	8.303	8.400	8.491	8.575	8.654	8.728	8.798	8.864	9.148	9.377	9.568	9.732	9.875	10.00
12	7.731	7.853	7.964	8.066	8.159	8.246	8.327	8.402	8.473	8.539	8.603	8.875	9.094	9.277	9.434	9.571	9.693
13	7.548	7.665	7.772	7.870	7.960	8.043	8.121	8.193	8.262	8.326	8.387	8.648	8.859	9.035	9.187	9.318	9.436
14	7.395	7.508	7.611	7.705	7.792	7.873	7.948	8.018	8.084	8.146	8.204	8.457	8.661	8.832	8.978	9.106	9.219
15	7.264	7.374	7.474	7.566	7.650	7.728	7.800	7.369	7.932	7.992	8.049	8.295	8.492	8.658	8.800	8.924	9.035
16	7.152	7.258	7.356	7.445	7.527	7.602	7.673	7.739	7.802	7.860	7.916	8.154	8.347	8.507	8.646	8.767	8.874
17	7.053	7.158	7.253	7.340	7.420	7.493	7.563	7.627	7.687	7.745	7.799	8.031	8.229	8.377	8.511	8.630	8.735
18	6.968	7.070	7.163	7.247	7.325	7.398	7.465	7.528	7.587	7.643	7.696	7.924	8.107	8.261	8.393	8.508	8.611
19	6.891	6.992	7.082	7.166	7.242	7.313	7.379	7.440	7.498	7.553	7.605	7.828	8.008	8.159	8.288	8.401	8.502
20	6.823	6.922	7.011	7.092	7.168	7.237	7.302	7.362	7.419	7.473	7.523	7.742	7.919	8.067	8.194	8.305	8.404
24	6.612	6.705	6.789	6.865	6.936	7.001	7.062	7.119	7.173	7.223	7.270	7.476	7.642	7.780	7.900	8.004	8.097
30	6.407	6.494	6.572	6.644	6.710	6.772	6.828	6.881	6.932	6.978	7.023	7.215	7.370	7.500	7.611	7.709	7.796
40	6.209	6.289	6.362	6.429	6.490	6.547	6.600	6.650	6.697	6.740	6.782	6.960	7.104	7.225	7.328	7.419	7.500
60	6.015	6.090	6.158	6.220	6.277	6.330	6.378	6.424	6.467	6.507	6.546	6.710	6.843	6.954	7.050	7.133	7.207
120	5.827	5.897	5.959	6.016	6.069	6.117	6.162	6.204	6.244	6.281	6.316	6.467	6.588	6.689	6.776	6.852	6.919
inf	5.645	5.709	5.766	5.818	5.866	5.911	5.952	5.990	6.026	6.060	6.092	6.228	6.338	6.429	6.507	6.575	6.636

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df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20
1	180.1	270.1	328.5	371.2	404.4	431.6	454.4	474.0	491.1	506.3	520.0	532.4	543.6	554.0	563.6	580.9	596.0
2	19.93	26.97	31.60	35.02	37.73	39.95	41.83	43.46	44.89	46.16	47.31	48.35	49.30	50.17	50.99	52.45	53.74
3	10.55	13.50	15.45	16.91	18.06	19.01	19.83	20.53	21.15	21.70	22.20	22.66	23.08	23.46	23.82	24.46	25.03
4	7.916	9.814	11.06	11.99	12.74	13.35	13.88	14.33	14.74	15.10	15.42	15.72	15.99	16.24	16.48	16.90	17.28
5	6.751	8.196	9.141	9.847	10.41	10.88	11.28	11.63	11.93	12.21	12.46	12.69	12.90	13.09	13.27	13.60	13.89
6	6.105	7.306	8.088	8.670	9.135	9.522	9.852	10.14	10.40	10.63	10.83	11.02	11.20	11.36	11.51	11.78	12.02
7	5.699	6.750	7.429	7.935	8.339	8.674	8.961	9.211	9.433	9.632	9.812	9.977	10.13	10.27	10.40	10.64	10.85
8	5.420	6.370	6.981	7.435	7.797	8.097	8.354	8.578	8.777	8.955	9.117	9.265	9.401	9.527	9.644	9.857	10.04
9	5.218	6.096	6.657	7.074	7.405	7.680	7.915	8.120	8.303	8.466	8.614	8.749	8.874	8.990	9.097	9.292	9.465
10	5.065	5.888	6.412	6.800	7.109	7.365	7.584	7.775	7.944	8.096	8.234	8.360	8.476	8.583	8.683	8.865	9.026
11	4.945	5.727	6.222	6.588	6.878	7.119	7.325	7.505	7.664	7.807	7.937	8.055	8.164	8.265	8.359	8.530	8.682
12	4.849	5.597	6.068	6.416	6.693	6.922	7.118	7.288	7.439	7.575	7.697	7.810	7.914	8.009	8.099	8.261	8.405
13	4.770	5.490	5.943	6.277	6.541	6.760	6.947	7.111	7.255	7.384	7.502	7.609	7.708	7.800	7.886	8.040	8.178
14	4.704	5.401	5.838	6.160	6.414	6.626	6.805	6.962	7.101	7.225	7.338	7.442	7.537	7.625	7.707	7.856	7.988
15	4.647	5.325	5.750	6.061	6.308	6.511	6.685	6.837	6.971	7.091	7.200	7.300	7.392	7.477	7.556	7.699	7.827
16	4.599	5.261	5.674	5.977	6.216	6.413	6.582	6.729	6.859	6.976	7.081	7.178	7.267	7.349	7.426	7.566	7.689
17	4.557	5.205	5.608	5.903	6.136	6.329	6.493	6.636	6.763	6.876	6.979	7.072	7.159	7.239	7.314	7.449	7.569
18	4.521	5.156	5.550	5.839	6.067	6.255	6.415	6.554	6.678	6.788	6.888	6.980	7.064	7.142	7.215	7.347	7.464
19	4.488	5.113	5.500	5.783	6.005	6.189	6.346	6.482	6.603	6.711	6.809	6.898	6.981	7.057	7.128	7.257	7.372
20	4.460	5.074	5.455	5.732	5.951	6.131	6.285	6.418	6.537	6.642	6.738	6.826	6.907	6.981	7.051	7.177	7.289
24	4.371	4.955	5.315	5.577	5.783	5.952	6.096	6.221	6.332	6.431	6.520	6.602	6.677	6.747	6.812	6.930	7.034
30	4.285	4.841	5.181	5.428	5.621	5.780	5.914	6.031	6.135	6.227	6.310	6.387	6.456	6.521	6.581	6.691	6.788
40	4.202	4.731	5.053	5.284	5.465	5.614	5.739	5.848	5.944	6.030	6.108	6.179	6.244	6.304	6.360	6.461	6.550
60	4.122	4.625	4.928	5.146	5.316	5.454	5.571	5.673	5.762	5.841	5.913	5.979	6.039	6.094	6.146	6.239	6.321
120	4.045	4.523	4.809	5.013	5.172	5.301	5.410	5.504	5.586	5.660	5.726	5.786	5.842	5.893	5.940	6.025	6.101
inf	3.970	4.424	4.694	4.886	5.033	5.154	5.255	5.341	5.418	5.485	5.546	5.602	5.652	5.699	5.742	5.820	5.889

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	596.0	609.5	621.7	632.6	642.7	652.0	660.6	668.5	676.0	683.0	689.6	717.8	740.2	758.8	774.5	788.2	800.3
2	53.74	54.89	55.92	56.86	57.73	58.52	59.26	59.95	60.59	61.19	61.76	64.19	66.13	67.74	69.10	70.29	71.35
3	25.03	25.54	26.00	26.42	26.80	27.15	27.48	27.79	28.07	28.34	28.60	29.68	30.55	31.27	31.88	32.42	32.90
4	17.28	17.61	17.91	18.19	18.44	18.68	18.89	19.09	19.28	19.46	19.63	20.36	20.93	21.42	21.83	22.18	22.50
5	13.89	14.14	14.38	14.59	14.79	14.96	15.13	15.29	15.44	15.58	15.71	16.27	16.72	17.09	17.41	17.69	17.94
6	12.02	12.23	12.43	12.61	12.77	12.92	13.06	13.19	13.32	13.43	13.54	14.02	14.40	14.71	14.98	15.21	15.43
7	10.85	11.03	11.21	11.36	11.50	11.64	11.76	11.88	11.99	12.09	12.18	12.60	12.93	13.21	13.44	13.65	13.84
8	10.04	10.22	10.37	10.51	10.64	10.76	10.87	10.97	11.07	11.16	11.25	11.63	11.93	12.18	12.39	12.58	12.75
9	9.465	9.620	9.761	9.890	10.01	10.12	10.22	10.32	10.41	10.49	10.58	10.92	11.20	11.43	11.63	11.80	11.96
10	9.026	9.170	9.302	9.422	9.532	9.635	9.730	9.820	9.904	9.983	10.06	10.38	10.64	10.86	11.04	11.20	11.35
11	8.682	8.818	8.941	9.055	9.159	9.256	9.345	9.430	9.509	9.583	9.654	9.957	10.20	10.41	10.59	10.74	10.88
12	8.405	8.534	8.652	8.759	8.858	8.950	9.036	9.116	9.191	9.262	9.328	9.617	9.850	10.04	10.21	10.36	10.49
13	8.178	8.302	8.414	8.516	8.611	8.699	8.781	8.857	8.929	8.997	9.061	9.337	9.560	9.747	9.907	10.05	10.17
14	7.988	8.107	8.215	8.314	8.404	8.489	8.568	8.641	8.710	8.775	8.837	9.103	9.317	9.497	9.652	9.787	9.907
15	7.827	7.942	8.046	8.141	8.229	8.311	8.387	8.458	8.524	8.587	8.647	8.904	9.111	9.285	9.434	9.565	9.680
16	7.689	7.800	7.901	7.994	8.078	8.158	8.231	8.300	8.365	8.425	8.483	8.733	8.933	9.102	9.247	9.373	9.486
17	7.569	7.677	7.775	7.865	7.948	8.024	8.096	8.163	8.226	8.285	8.341	8.583	8.779	8.943	9.084	9.206	9.316
18	7.464	7.570	7.665	7.753	7.833	7.908	7.978	8.043	8.104	8.162	8.217	8.452	8.643	8.803	8.940	9.061	9.167
19	7.372	7.474	7.568	7.653	7.732	7.805	7.873	7.937	7.996	8.053	8.106	8.337	8.523	8.679	8.813	8.931	9.036
20	7.289	7.390	7.481	7.565	7.642	7.713	7.780	7.842	7.901	7.956	8.008	8.234	8.416	8.569	8.700	8.815	8.917
24	7.034	7.128	7.213	7.291	7.362	7.429	7.491	7.549	7.603	7.655	7.704	7.914	8.083	8.226	8.348	8.455	8.551
30	6.788	6.875	6.954	7.026	7.093	7.154	7.212	7.265	7.316	7.364	7.409	7.603	7.760	7.893	8.006	8.105	8.193
40	6.550	6.631	6.704	6.770	6.832	6.889	6.942	6.991	7.038	7.082	7.123	7.302	7.447	7.568	7.672	7.763	7.845
60	6.321	6.396	6.462	6.523	6.580	6.632	6.681	6.726	6.769	6.808	6.846	7.010	7.143	7.252	7.347	7.431	7.504
120	6.101	6.169	6.230	6.286	6.337	6.385	6.428	6.470	6.508	6.545	6.580	6.728	6.846	6.946	7.032	7.107	7.173
inf	5.889	5.951	6.006	6.057	6.103	6.146	6.186	6.223	6.258	6.291	6.322	6.454	6.561	6.649	6.725	6.792	6.850

Alpha = 0.001

df	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20
1	900.3	1351	1643	1856	2022	2158	2272	2370	2455	2532	2600	2662	2718	2770	2818	2904	2980
2	44.69	60.42	70.77	78.43	84.49	89.46	93.67	97.30	100.5	103.3	105.9	108.2	110.4	112.3	114.2	117.4	120.3
3	18.28	23.32	26.65	29.13	31.11	32.74	34.12	35.33	36.39	37.34	38.20	38.98	39.69	40.35	40.97	42.07	43.05
4	12.18	14.99	16.84	18.23	19.34	20.26	21.04	21.73	22.33	22.87	23.36	23.81	24.21	24.59	24.94	25.58	26.14
5	9.714	11.67	12.96	13.93	14.71	15.35	15.90	16.38	16.81	17.18	17.53	17.85	18.13	18.41	18.66	19.10	19.51
6	8.427	9.96	10.97	11.72	12.32	12.83	13.26	13.63	13.97	14.27	14.54	14.79	15.01	15.22	15.42	15.78	16.09
7	7.648	8.930	9.768	10.40	10.90	11.32	11.68	11.99	12.27	12.52	12.74	12.95	13.14	13.32	13.48	13.78	14.04
8	7.130	8.250	8.978	9.522	9.958	10.32	10.64	10.91	11.15	11.36	11.56	11.74	11.91	12.06	12.21	12.47	12.70
9	6.762	7.768	8.419	8.906	9.295	9.619	9.897	10.14	10.36	10.55	10.73	10.89	11.03	11.18	11.30	11.54	11.75
10	6.487	7.411	8.006	8.450	8.804	9.099	9.352	9.573	9.769	9.946	10.11	10.25	10.39	10.52	10.64	10.85	11.03
11	6.275	7.136	7.687	8.098	8.426	8.699	8.933	9.138	9.319	9.482	9.630	9.766	9.892	10.01	10.12	10.31	10.49
12	6.106	6.917	7.436	7.821	8.127	8.383	8.601	8.793	8.962	9.115	9.254	9.381	9.498	9.606	9.707	9.891	10.06
13	5.970	6.740	7.231	7.595	7.885	8.126	8.333	8.513	8.673	8.817	8.948	9.068	9.178	9.281	9.376	9.550	9.704
14	5.856	6.594	7.062	7.409	7.685	7.915	8.110	8.282	8.434	8.571	8.696	8.809	8.914	9.012	9.103	9.267	9.414
15	5.760	6.470	6.920	7.252	7.517	7.736	7.925	8.088	8.234	8.365	8.483	8.592	8.693	8.786	8.872	9.030	9.170
16	5.678	6.365	6.799	7.119	7.374	7.585	7.766	7.923	8.063	8.189	8.303	8.407	8.504	8.593	8.676	8.828	8.963
17	5.608	6.275	6.695	7.005	7.250	7.454	7.629	7.781	7.916	8.037	8.148	8.248	8.342	8.427	8.508	8.654	8.784
18	5.546	6.196	6.604	6.905	7.143	7.341	7.510	7.657	7.788	7.906	8.012	8.110	8.199	8.283	8.361	8.502	8.628
19	5.492	6.127	6.525	6.817	7.049	7.242	7.405	7.549	7.676	7.790	7.893	7.988	8.075	8.156	8.232	8.369	8.491
20	5.444	6.065	6.454	6.740	6.966	7.154	7.313	7.453	7.577	7.688	7.788	7.880	7.966	8.044	8.118	8.251	8.370
24	5.297	5.877	6.238	6.503	6.712	6.884	7.031	7.159	7.272	7.374	7.467	7.551	7.629	7.701	7.768	7.890	7.999
30	5.156	5.698	6.033	6.278	6.470	6.628	6.763	6.880	6.984	7.077	7.162	7.329	7.310	7.375	7.437	7.548	7.647
40	5.022	5.528	5.838	6.063	6.240	6.386	6.509	6.616	6.711	6.796	6.872	6.942	7.007	7.067	7.122	7.223	7.312
60	4.894	5.365	5.653	5.860	6.022	6.155	6.268	6.366	6.451	6.528	6.598	6.661	6.720	6.774	6.824	6.914	6.995
120	4.771	5.211	5.476	5.667	5.815	5.937	6.039	6.128	6.206	6.276	6.339	6.396	6.448	6.496	6.542	6.623	6.695
inf	4.654	5.063	5.309	5.484	5.619	5.730	5.823	5.903	5.973	6.036	6.092	6.144	6.191	6.234	6.274	6.347	6.411

df	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	2980	3047	3108	3163	3213	3260	3303	3343	3380	3415	3448	3589	3701	3794	3873	3941	4002
2	120.3	122.9	125.2	127.3	129.3	131.0	132.7	134.2	135.7	137.0	138.3	143.7	148.0	151.6	154.7	157.4	159.7
3	43.05	43.92	44.70	45.42	46.07	46.68	47.24	47.77	48.26	48.72	49.16	51.02	52.51	53.75	54.81	55.72	56.53
4	26.14	26.65	27.10	27.51	27.89	28.24	28.57	28.88	29.16	29.43	29.68	30.78	31.65	32.37	32.98	33.52	34.00
5	19.51	19.86	20.19	20.48	20.75	21.01	21.24	21.46	21.66	21.86	22.03	22.82	23.45	23.97	24.41	24.80	25.15
6	16.09	16.38	16.64	16.87	17.08	17.28	17.47	17.64	17.81	17.96	18.10	18.73	19.22	19.64	20.00	20.31	20.58
7	14.04	14.29	14.50	14.70	14.88	15.05	15.20	15.35	15.49	15.62	15.74	16.27	16.69	17.04	17.35	17.61	17.85
8	12.70	12.91	13.09	13.26	13.42	13.57	13.71	13.84	13.96	14.07	14.18	14.64	15.01	15.32	15.59	15.82	16.02
9	11.75	11.93	12.10	12.25	12.39	12.53	12.65	12.77	12.87	12.97	13.07	13.49	13.82	14.10	14.34	14.55	14.74
10	11.03	11.20	11.36	11.50	11.63	11.75	11.87	11.97	12.07	12.16	12.25	12.63	12.94	13.20	13.42	13.61	13.78
11	10.49	10.65	10.79	10.92	11.04	11.16	11.26	11.35	11.45	11.53	11.62	11.97	12.25	12.49	12.70	12.88	13.04
12	10.06	10.20	10.34	10.46	10.57	10.68	10.78	10.87	10.96	11.03	11.11	11.44	11.71	11.94	12.13	12.29	12.45
13	9.704	9.843	9.969	10.09	10.19	10.29	10.39	10.47	10.55	10.63	10.70	11.01	11.27	11.48	11.66	11.82	11.97
14	9.414	9.546	9.666	9.776	9.878	9.972	10.06	10.14	10.22	10.30	10.37	10.66	10.91	11.11	11.28	11.43	11.57
15	9.170	9.296	9.411	9.517	9.613	97.03	9.788	9.867	9.940	10.01	10.08	10.37	10.59	10.79	10.96	11.10	11.23
16	8.963	9.084	9.194	9.295	9.388	9.475	9.556	9.631	9.702	9.769	9.833	10.11	10.34	10.52	10.68	10.82	10.95
17	8.784	8.900	9.007	9.104	9.194	9.277	9.355	9.429	9.497	9.562	9.623	9.888	10.10	10.29	10.44	10.58	10.70
18	8.628	8.741	8.844	8.938	9.025	9.106	9.181	9.251	9.318	9.381	9.440	9.696	9.904	10.08	10.23	10.36	10.48
19	8.491	8.601	8.701	8.792	8.876	8.955	9.028	9.096	9.161	9.221	9.279	9.528	9.730	9.899	10.04	10.17	10.29
20	8.370	8.477	8.574	8.663	8.745	8.821	8.892	8.959	9.021	9.081	9.137	9.379	9.575	9.740	9.881	10.01	10.12
24	7.999	8.097	8.185	8.267	8.342	8.411	8.476	8.537	8.594	8.648	8.700	8.921	9.100	9.250	9.380	9.494	9.596
30	7.647	7.735	7.816	7.890	7.958	8.021	8.080	8.135	8.188	8.237	8.283	8.484	8.647	8.783	8.901	9.004	9.096
40	7.312	7.393	7.466	7.533	7.594	7.651	7.704	7.754	7.801	7.845	7.887	8.067	8.214	8.337	8.442	8.535	8.618
60	6.995	7.067	7.133	7.193	7.248	7.299	7.347	7.392	7.433	7.473	7.510	7.671	7.802	0.924	8.005	8.088	8.161
120	6.695	6.760	6.818	6.872	6.921	6.966	7.008	7.048	7.085	7.121	7.153	7.296	7.411	7.507	7.590	7.662	7.726
inf	6.411	6.469	6.520	6.568	6.611	6.651	6.689	6.723	6.756	6.787	6.816	6.941	7.041	7.124	7.196	7.259	7.314