ESL Ex 10.2: Prove result 10.16 – that is – the minimizer of the population version of the AdaBoost criterion is one half of the log-odds

$$(10.16) \quad f^*(x) = \underset{f(x)}{\operatorname{argmin}}\; E_{Y|x}\left(e^{-Yf(x)}\right)$$

$$= \frac{1}{2} \ln\left(\frac{P(Y=1\,|x)}{P(Y=-1\,|x)}\right)$$

$$\frac{\partial E_{Y|x}\left(e^{-Y\cdot f(x)}\right)}{\partial f(x)} = E_{Y|x}\left(\frac{\partial e^{-Yf(x)}}{\partial f(x)}\right) = \underline{E_{Y|x}\left(-Y\cdot e^{-Yf(x)}\right) = 0}$$

$$Y = \begin{cases} -1 & \text{with } P(Y=-1\,|x) \\ 1 & P(Y=1\,|x) \end{cases}$$

$$E_{Y|x}\left(-Y\cdot e^{-Yf(x)}\right) = -(-1)\cdot e^{f(x)}\cdot P(Y=-1|x) - 1\cdot e^{-f(x)}\cdot P(Y=1\,|x) = 0$$

$$e^{f(x)}\left(e^{f(x)}\cdot P(Y=-1|x) - e^{-f(x)}\cdot P(Y=1\,|x)\right) = 0$$

$$e^{2f(x)}P(Y=-1\,|x) = P(Y=1\,|x)$$

$$\underline{\underline{f(x) = \frac{1}{2}\ln\frac{P(Y=1\,|x)}{P(Y=-1\,|x)}}}$$