Corrections to Karr: Probability

Chapter 1

page 27, line 12↑ The right hand side of the inequality should be changed from

$$\leq P(A_k)$$

to

$$\leq \inf_{n \geq k} P(A_n)$$

Chapter 2

page 44, Example 2.4 The left hand side of the equations should be

$$\{\mathbf{1}_A \in B\}$$

page 49, Theorem 2.16 Add that the random values could have values in $\bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$.

Chapter 3

page 82, line 2↓ Should read: ...1 for each n.

page 91, Proposition 3.37 One needs to say $t_0 = 0$. In the proof on the next page, $k_0 = 0$.

page 93, line 9↑ The function g might better be defined from $\mathbb{R}^k$ to $\mathbb{R}^k$.

Chapter 4

page 105, lines 3-4↓of the proof of Theorem 4.8 Read as: for each m and $k \geq m$,

$$X_k \geq Z_m \geq (Y - \epsilon) \cdots$$

page 121, line 2↓ Should be $E[X^2] \geq E[X]^2$

page 121, proof of Theorem 4.38 To prove that $X + Y \in L^p$ when X and Y are in L^p, one may use the inequality (prove it!)

$$|X + Y|^p \leq 2^p \left(|X|^p + |Y|^p\right)$$

page 122, line 4↓of the proof of Theorem 4.40 The last part of the line should be

$$\geq g(a) P(X \geq a)$$

Chapter 5

page 138, line 4↓of the proof of Theorem 5.8 and page 139, line 6↓ "Sufficiency" and "Necessity" should be interchanged. Thus, the first part of the proof proves "⇒", while the second part proves "⇐"

page 144, last three lines of the proof of Theorem 5.17 These lines should read:

well, choose $\delta > 0$ such that whenever $P(A) < \delta$, $\sup_{n \leq N} E\left[|X_n - X|; A\right] < \epsilon$ and $E\left[|X|; A\right] < \epsilon$. Then, whenever ...

Note that $\sup_n E\left[|X_n - X|; A\right] < \epsilon$ because we already have required $E\left[|X_n - X|\right] < \epsilon$ for $n \geq N$.

The following Lemma is used in the proof:

Lemma: Let $X \in L^1$. Then for each $\epsilon > 0$ there is a $\delta > 0$ such that

$$P(A) < \delta \Rightarrow E\left[|X|; A\right] < \epsilon$$

page 154, line 6↓ The line should read:

$$= -x\sqrt{npq} + \frac{1}{2}x^2 q - x^2 q + \cdots$$

page 154, line 7↓ The line should read:

$$+x\sqrt{npq} + \frac{1}{2}x^2 p - x^2 p - \frac{1}{2}x^3 p\sqrt{\frac{p}{nq}}$$

page 155, line 5↑ Read: *It suffices, by Problem 5.7, to show that...*

Chapter 6

page 170, line 5↑ In the second expression should be $d\tilde{G}(x)$ instead of $dG(x)$.

page 172, proof of Sufficiency This proof is not correct. Use instead Theorem 6.17.

page 178, line 2↑ Should read: ...while also $F_{n'}(r') \to \tilde{G}(r')$, $F_{n'}(r'') \to \tilde{G}(r'')$, and...

Chapter 7

page 184, line 9↓ The last part should be

$$\bigcup_{k=1}^{n} \{|S_k| > \epsilon\}$$

page 185, line 6↓ Should be

$$P(A) \geq \frac{E[S_n^2] - \epsilon^2}{\cdots} \geq \cdots$$

page 186, Equation (7.5) Should be

$$|\sum_{k=1}^{\infty} E[X_k^c]| < \infty$$

page 187, line 10↓ At the end should in the numerator be $(2c+\epsilon)^2$, and the sum in the denominator should go from $k = n+1$.

page 187, line 15↓ $|S_{n+k} - S_n|$ should be $|S^*_{n+k} - S^*_n|$

page 188, line 1-2↑ The two $\geq$ should be $>$. The $+1$ at the end should be deleted. Refer to Exercise 4.9.

page 192, line 9↑ Should start by $(s^2_{j_n}/\sigma_n^2)^{3/2}$

page 193, Equation (7.16) Should be $|X_k|$.

page 194, Equation (7.19) The sup should be over $x \in \mathbb{R}$.

page 195, lines 6-10↓ Replace these lines by the following:

For all h_1, h_2 and all x we have by (7.19),

$$|f(x+h_1) - f(x+h_2) - f'(x)(h_1 - h_2) - \frac{1}{2}f''(x)(h_1^2 - h_2^2)| \leq g(h_1) + g(h_2)$$

We will use this with $x = U_n^{(k)}/\sigma_n$, $h_1 = X_k/\sigma_n$, $h_2 = Z_k/\sigma_n$.

Note first that by independence of $U_n^{(k)}, X_k, Z_k$ we have

$$E\left[f'\left(\frac{U_n^{(k)}}{\sigma_n}\right)\frac{X_k - Z_k}{\sigma_n}\right] = 0$$
$$E\left[f''\left(\frac{U_n^{(k)}}{\sigma_n}\right)\frac{X_k^2 - Z_k^2}{\sigma_n^2}\right] = 0$$

Hence for each term in line 5↓,

$$\begin{aligned}
& \left|E\left[f\left(\frac{U_n^{(k)} + X_k}{\sigma_n}\right) - f\left(\frac{U_n^{(k)} + Z_k}{\sigma_n}\right)\right]\right| \\
= & \left|E\left[f\left(\frac{U_n^{(k)} + X_k}{\sigma_n}\right) - f\left(\frac{U_n^{(k)} + Z_k}{\sigma_n}\right)\right.\right. \\
- & \left.\left. f'\left(\frac{U_n^{(k)}}{\sigma_n}\right)\frac{X_k - Z_k}{\sigma_n} - \frac{1}{2}f''\left(\frac{U_n^{(k)}}{\sigma_n}\right)\frac{X_k^2 - Z_k^2}{\sigma_n^2}\right]\right| \\
\leq & E\left[g\left(\frac{X_k}{\sigma_n}\right)\right] + E\left[g\left(\frac{X_k}{\sigma_n}\right)\right]
\end{aligned}$$

page 197, line 2↑ Should be "equality" instead of "inequality"

page 198, line 2↓ Should be a dx to the left.

page 198, Equations (7.27) and (7.28) Delete the "a.s."

page 199, lines 10-12↓ The right hand sides of the inequalities should all be multiplied by $(1+\epsilon)$

page 199, line 7↑ The expression in $[\cdot]$ is not correct. Instead we have

$$\frac{n_{k+1}}{n_k} \to 1+\epsilon \text{ as } k \to \infty$$

page 199, line 3↑ Delete the "a.s."

page 200, line 7↓ The lower bound on $P(A_k)$ is not correct (see lecture)