1. Give an example of a sequence of events $A_1, A_2, ...$ such that

$$\liminf_n A_n = \emptyset \text{ (empty)},$$

$$\limsup_n A_n = \Omega \text{ (sample space)}.$$

2. Let X and Y be independent, absolutely continuous random variables with densities $f_X(x)$ and $f_Y(y)$, respectively. Suppose that these densities are bounded:

$$f_X(x) \le A, \quad f_Y(y) \le B$$

for some positive A and B for any x and y. Prove that

$$f_{X+Y}(t) \le \min\{A, B\}$$

for any t.

3. Let X and Y be iid absolutely continuous random variables. Denote by f_{X-Y} the probability density function of the difference $X - Y$. Prove that

$$f_{X-Y}(t) \le f_{X-Y}(0)$$

for any t.

4. Let $X_1, X_2, ...$ be iid random variables with finite first absolute moment and positive expectation, $S_n = X_1 + ... + X_n$. Prove that

$$\lim_{n\to\infty} P(S_n \le x) = 0$$

for any x.

5. Let $\varphi(t)$ be a characteristic function. Show that $\Re\varphi(t)$ is also a characteristic function. Can $\Im\varphi(t)$ be a characteristic function? ($\Re z$ and $\Im z$ are the real and the imaginary part of z, respectively).