

1. Let

$$X_n \xrightarrow{L^2} X, \quad n \rightarrow \infty.$$

Prove that

$$\text{Var}X_n \rightarrow \text{Var}X, \quad n \rightarrow \infty.$$

2. $X_\lambda \sim \text{Pois}(\lambda)$, $\lambda > 0$. Use characteristic functions to prove that

$$\frac{X_\lambda - \lambda}{\sqrt{\lambda}} \xrightarrow{d} N(0, 1)$$

as $\lambda \rightarrow \infty$.

3. Let X_1, X_2, \dots be a sequence of iid random variables with a finite variance $\text{Var}X_n < \infty$. Prove that for any x , the limit

$$\lim_{n \rightarrow \infty} P(X_1 + \dots + X_n \leq x)$$

can take only three values. Which values? Find conditions under which the limit takes each of these three values.